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note e commenti

Iterated modal propositions in scholastic logic: the S5 stance*

LYNN CATES**

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Prologue

In the history of philosophy there are many cases where one asserts certain propositions. In looking back at such sets of propositions the trained historian of philosophy can see the conclusion, or conclusions, to which the proposer of those propositions should have been committed. Note well that I say "should have been committed" for often it is the case—sometimes frustratingly so—that the proposer holding those propositions had a blind spot regarding the obvious conclusion entailed by them. To take a simple but well known case: John Locke seems to be oblivious to the fact that his philosophical views lead straight to subjective idealism. But for the even mildly sophisticated philosopher of today, the entailment is obvious: one cannot simultaneously hold that I) all knowledge comes from experience[1], II) we have no experience with material substance, and III) we can know that there is an extramental material reality. Indeed, George Berkeley was quick to note the correct conclusion to Locke's premises.

Another kind of blindness—less culpable that the first—among philosophers

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** 1401 E. Rundberg #357, Austin Community College, Austin, Texas 78753, USA

[1] Of course, one could offer here that Locke made an exception in his empirical epistemology for knowledge of God and knowledge of self and could have made a similar exception for knowledge of material substance. But what, then, remains of his empirical claim that "Whence has [the mind] all the materials of reason and knowledge? To this I answer, in one word, from experience."? (*Essay Concerning the Human Understanding*, Book II, Chapter1, Paragraph 2, *The Empiricists*, Richard Taylor. New York: Doubleday, 1974.(For such a "limited empiricism" has a technical name: rationalism.

living before the contemporary analytical movement was to hold a certain set of propositions which entailed a certain conclusion and yet the conclusion was not drawn simply because a single catalyst proposition, or bit of technical apparatus, was missing. So, for example, Epicurus could not draw the conclusion which is the negation of "the Problem of Evil" because he lacked one or more of the propositions of Leibniz expressed in his *Theodicy*, while William of Ockham lacked the propositional calculus developed in the twentieth century.

One fascinating academic exercise is examining the logical works of mediaeval logicians for modal propositions—whether these refer to logical, theological, or metaphysical modes—or propositions concerned with modal propositions—again, whether these refer to logical, theological, or metaphysical modes. In particular, one would like to discover these logicians' views about iterated modal propositions like those used in modern logical systems, particularly S5, characterized by the axiom schema

$M\alpha \supset LM\alpha$.

I claim that, the serious historian of logic cannot escape the fact that propositions were proposed by mediaeval logicians which could have encouraged their acceptance of the modal system S5, if their propositions did not actually entail them. Clearly, I am not claiming that, say, a fourteenth century logician accepted the axiom schema of S5 in all its particulars, if for no other reason than that he lacked the technical apparatus—here, the modern propositional calculus—to do so. Rather, I am claiming that, given some mediaeval logicians' background assumptions concerning logic, ontology, and theology, they were driven toward an S5 stance, if you will. Indeed, I claim that some of those who adopt an "S5 stance" would have been committed to the axiom schema of that system had it been available in modern form.

At least two conditions must obtain before a higher order modal system like S5 can come to be: I) alethic modal words should be understood as operators rather than merely as predicates; II) modal words must refer to synchronic alternatives—'possible worlds'—rather than statistical frequencies. Additionally, it is desirable to undergird such a modal system with a coherent ontology. In this paper I shall give a brief overview of the development of the S5 stance.

I. Sherwood and the move toward operators

Apparently, William of Sherwood (ca. 1200-10 - ca. 1266-72) fulfilled condition I in the thirteenth century. In solving the sophism "Only necessaries necessarily are true"[2] William separates the analysis into two cases, the latter of which

[2] WILLIAM OF SHERWOOD, *Syncategoremata Magistri Guillelmi de Shireswode*, J. Reginald O' Donnell, *Mediaeval Studies*, vol. 3 (1941) p. 74.
"*[S]ola necessaria necessario sunt vera*".
(All translations mine unless otherwise noted).

breaks apart into two subcases, the latter of which breaks apart further into two sub-subcases.

In the first case 'necessarily' is treated as a predicate. In the second case, however, William conceives 'necessarily' as an operator whose scope is either I) the entire remaining proposition or, II) the entire remaining proposition except 'only'[3]. Thus the first subcase amounts to: "Necessarily, only necessary [propositions] are true." Symbolically, this could be thought of as $L((L\alpha \supset \alpha) \bullet (\sim L\alpha \supset \sim\alpha))$. Evidently, this is a significant move for the development of a higher order modal logic for now conditions are ripe for the stance mentioned above. Nor can I locate any earlier examples of such conditions.

II. Possible worlds from Lombard to Scotus

For the second condition to obtain, it is desirable that one engage in ontological or theological speculation concerned with counterfactuals about the way things are. (Hence my insistence above that one consider not only modal logical propositions but ontological and theological modes as well.) This stipulation seems to have been fulfilled at least as early as the twelfth century by Peter Lombard in his *Sententiae*. He asks:

> "Whether [God] can make [something] in another or better way than He makes it. After this it is to be considered whether [God] can make that which He makes in another or better way than he makes it. Reply. If the manner of the artist's work is referred to, [then] neither in another nor better [way] could it be. For He cannot make something in another or better way than He makes it, i.e., by another or greater wisdom, for He can make nothing more wisely than he makes it. If, however, the condition of the thing itself which God makes is referred to, we claim that the condition can be both other and better [than it is]. And according to this [distinction], it can be granted that what He makes He can make better and in another way than He makes it, because he can make something in a better way of existing, and something in another [way of existing]....Therefore, of those things which He makes, God can make some in another better way, some in another equally good way, [and] even some in a less good way than He makes [them]. So that, however, the "way" refers to the quality of the work, that is, of the creature, not the wisdom of the Creator"[4].

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[3] The sub-subcases need not concern us.

[4] P. LOMBARD, *In Quatuor Libros Sententiarum*, Frankfurt: Unveranderter, 1967, I, d. 44.
"*Utrum alio vel meliori modo possit facere quam facit. Post haec considerandum est utrum alio modo vel meliori quam facit, possit ea facere quae facit.*
Responsio. Si modus operationis ad sapientiam opificis referatur, nec alius, nec melior esse potest. Non enim potest facere aliquid aliter vel melius quam facit, id est alia sapientia vel maiori sapientia: nihil enim sapientius potest facere quam facit. Si vero referatur modus ad

Notice that in dealing with the second part of this question, one arrives at three broad grades for how the world could have been but is not—better, equally good, or not as good. When, however, the other part of the question is considered (or when these broad groups are sorted into finer partitions) one finds many alternate ways in which the world could have been made.

One can go further here: the ways in which the world could be and are not will never come about. And such an account marks a departure from both the statistical frequency account of modal terms, particularly possibility, and the diachronic account. Hence is born the notion of synchronous possibilities—possible worlds. Granted, it would be rash to try to equate these one to one with the possible worlds semantics of Saul Kripke; nevertheless, it is clear that, to some extent, his thought is tending in that direction. And however he seems to conceive of these worlds in their particular aspects, Lombard seems to explain little about them.

In the next century, Thomas Aquinas considers something like possible worlds in dealing with God's knowledge of what is not:

> "Now a certain difference is to be noted in the consideration of those things that are not actual. For though some of them may not be in act now, still they were, or they will be; and God is said to know all these with the knowledge of vision: for since God's act of understanding, which is His being, is measured by eternity; and since eternity is without succession, comprehending all time, the present glance of God extends over all time, and to all things which exist in any time, as to objects present to him. But there are other things in God's power, or the creature's, which nevertheless are not, nor will be, nor were; and as regards these He is said to have the knowledge, not of vision, but of simple intelligence. This is so called because the things we see around us have distinct being outside the seer"[5].

Clearly, such "things" which never shall be actualized cannot refer to statistical frequencies, for those must be actualized at some time. Rather they refer to alternate ways the world could be—possible worlds, if you will, or something like them.

Unfortunately, Thomas says little about such Divine cognitive contents, dismissing speculation thereon with: "*I answer that,* The created intellect, in seeing the divine essence, does not see in it all that God does or can do"[6]. But if the created intellect cannot penetrate the Divine realm of possibles, no one can blame Thomas for his silence about their nature.

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rem ipsam quam facit Deus, dicimus quia et alius et melior potest esse modus. Et secundum hoc concedi potest quia ea quae facit, potest facere melius et aliter quam facit, quia potest quibusdam meliorem modum exsistendi praestare, et quibusdam alium.... Potest igitur Deus eorum quae facit quaedam alio modo meliori, quaedam alio modo aeque bono, quaedam etiam minus bono facere quam facit: ut tamen modus referatur ad qualitatem operis, id est creaturae, non ad sapientiam Creatoris".

[5] St. Thomas Aquinas, *Summa Theologica*, I. q 14. a 9.

[6] St. Thomas Aquinas, op. cit. I. q 12. a 8.

Scotus, at the end of the century, is the pivotal figure in developing an account of synchronous possibilities in the Divine mind and a subsequent account of logical possibility. In the *Tractatus de Primo Principio*, Scotus says: "Here I do not call contingent whatever is not necessary nor sempiternal, but [that] of which the opposite can become [the case] when that [first thing] exists"[7]. As Knuutila points out, with this account of possibility, Scotus rejects the older statistical frequency model of possibility[8].

But which states of affairs count as possible for Scotus? He says: "[A]nything whatsoever is to be posited to be possible, whose impossibility [is] not evident..."[9]. And so now we have, by this definition, what modern logicians would refer to as logical possibility, that is, something counts as possible just in case its opposite does not entail a contradiction.

With Scotus' redefining of the possibles, we complete our second requirement for the development of an S5 stance. It remains now for someone explicitly to formulate a proposition like the axiom schema of S5, preferably as a result of thinking about the conditions needed in order to hold such possibles to be necessarily such, that is, necessarily possible.

III. Burleigh, Campsall and Lull

Historically, the fourteenth century is crucial for the development of the S5 stance. Around the beginning of the century, Walter Burleigh (ca. 1275 - ca. 1344) seems to reject propositions which seem remarkably similar to the characteristic axiom schemata of both S4 and S5. In solving the sophism, "The impossible can be true" Burleigh claims:

> "The impossible can be true. It is proved in this way. What can be possible, can be true; but the impossible can be possible; therefore, the impossible can be true. The antecedent is true; therefore, so is the consequent. [The sophism] is disproved thus. Everything which can be true, is possible; but the impossible can be true; therefore, the impossible is possible. The conclusion is false, therefore, [so is] one

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[7] J. Duns Scotus, *Tractatus de Primo Principio,* ed. and trans. Allan B. Wolter O.F.M. Chicago: Franciscan Herald Press, 1966. Chapter 4, Section 18. "*Non dico hic contingens quodcumque non est necessarium nec sempiternum, sed cuius oppositum posset fieri quando istud fit*".

[8] S. Knuutila, *Modal logic*, in *The Cambridge History of Later Medieval Philosophy*, Norman Kretzmann, Anthony Kenny, and Jan Pinborg (Cambridge: Cambridge University Press, 1982), p. 353. For an in depth account of Scotus' transition to possible worlds semantics see Knuutila, *Modalities in Medieval Philosophy*, (Routledge, London 1993) Ch. 4.

[9] J. Duns Scotus, *Ordinatio* in *Philosophical Writings*, trans. Allan B. Wolter O.F.M. Indianapolis: Hackett, 1987. I. d 2. q 1. s 2. a 2. "*[Q]uodlibet ponendum est esse possibile, cuius non apparet impossibilitas...*".

> of the premises. [But] the minor [premise is] not [false]. Therefore, the major [premise is]... Solution: The first [proposition] is ambiguous according to composition and division. In the sense of composition it is false and denotes that this is possible: "The impossible is true". In the sense of division it is again ambiguous since that which is impossible can be accepted for that which is impossible, and thus it is false, or [it can be accepted] for that which can be impossible, and thus it is true. And in this way the proof goes through. For the disproof, I claim (accepting the major [premise] according to that which is true) this conclusion, "The impossible is possible" does not follow. But this [conclusion], "That which can be possible is impossible" (and this is true) does [follow]"[10].

There are two crucial claims here. The first amounts to: "What can be impossible can be true". It seems that Burleigh wants to say that some proposition can be both possibly impossible and possibly true—that is, merely possible. Expressed in a formally stated object language this becomes: $M{\sim}M\alpha \bullet M\alpha$, where 'M' denotes the possibility operator and 'α' denotes some proposition.

If this is a correct formalization of his claim, it is easy to verify that this entails the denial of the characteristic axiom schema of S5 as follows:

{1} 1. $M{\sim}M\alpha \bullet M\alpha$	Assumption
{1} 2. $\sim LM\alpha \bullet M\alpha$	Def. of 'M' and double negation
{1} 3. $M\alpha \bullet \sim LM\alpha$	Commutation
{1} 4. $\sim(M\alpha \supset LM\alpha)$	Def. of '$\supset$', De Morgan's theorem, double negation.

Q. E. D.

The second crucial claim is: "What can be possible is impossible". This seems to amount to some proposition's being both possibly possible and yet being impossible. Stated formally, this would be: $MM\alpha \bullet \sim M\alpha$.

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[10] W. BURLEIGH, *De Puritate Artis Logicae Tractatus Longior,* (rpt. St. Bonaventure, N.Y.: Franciscan Institute, 1955) pp. 240-241.
"*Impossibile potest esse verum. Probatur sic. Quod potest esse possibile, potest esse verum; sed impossibile potest esse possibile, igitur impossibile potest esse verum. Antecedens est verum, igitur et consequens. Improbatur sic. Omne quod potest esse verum, est possibile; sed impossibile potest esse verum, igitur impossibile est possibile. Conclusio falsa, ergo aliqua praemissarum; non minor, ergo maior.*
Solutio. Prima est multiplex secundum compositionem et divisionem. In sensu compositionis est falsa, et denotatur, quod haec est possibilis: 'Impossibile est verum'. In sensu divisionis est ulterius multiplex, ex eo quod impossibile potest accipi pro eo, quod est impossibile, et sic est falsa, vel pro eo, quod potest impossibile, et sic est vera; et sic procedit probatio. Ad improbationem dico, accipiendo maiorem secundum quod vera est, non sequitur conclusio haec: "Impossibile est impossibile", sed ista: "Quod potest esse possibile est impossibile", et hoc est verum".
See also my paper, *A Note on Two Modal Propositions of Burleigh*, "Acta Philosophica" 8/I (1999).

It is easy to show formally in a modern derivational system that this entails the denial of the characteristic axiom schema of S4:[11]

{1} 1. MMα • ~Mα Assumption
{1} 2. ~(MMα ⊃ Mα) Def. of '⊃', De Morgan's theorem, double negation.
Q. E. D.

If I have correctly interpreted Burleigh's two propositions, evidently the denial of these two characteristic axiom schemata would follow. Hence, one could say he is opposed to the S5 stance or that his stance is anti-S5.

Does Burleigh have a good logical reason, based on his expressed views, for denying these axiom schemata? It seems he does. Notice that both require an iteration of a modal operator. Hence, if one explicitly rejects a meaningful iteration of such operators, one seems forced to deny these axiom schemata. And Burleigh does just this. He claims:

> "A modal proposition cannot be modified through the mode of anything, because then there would be two modes. And then, if it were the same mode, it would be redundant; if different modes, either it would be opposite in adjacency, or it would be redundant, as if one mode is useless compared to another [mode], as the possible is useless compared to the necessary"[12].

With this argument Burleigh must, in order to be consistent, rule out all higher order modal systems, insofar as each is characterized by an iterated modal proposition. That is, he must do so if and whenever he is presented with them.

A more sophisticated view is held by Richard of Campsall (ca. 1290—ca. 1355). He contends that:

> "Another principal [argument]: It does not follow of necessity: "Nothing which shall be necessary shall be impossible; therefore, of necessity nothing which shall be impossible shall be necessary", because the antecedent is true and the consequent false. The truth of the antecedent is obvious because any singular you please is true, and the falsity of the consequent is obvious because from it and another true [premise] a false [conclusion] follows by arguing thus: "Of necessity nothing impossible shall be necessary; "You have been a bishop" shall be impossible; therefore, of necessity, it shall not be necessary". The conclusion is false because

[11] The characteristic axiom schema of S4, properly, is Lα ⊃ LLα. Nevertheless, MMα ⊃ Mα is an equipollent form of it.

[12] *Ibidem*, p. 236.
"*Propositio modalis non potest modificari per modum alicuius, quia sic essent duo modi. Et tunc, si sit idem modus, erit nugatio. Si diversi modi, vel erit oppositum in adiecto vel erit nugatio, ut si unus modus sit in plus quam alius, sicut possibile est in plus quam necessarium*".

it is indeterminate whether it shall be necessary or not, and the minor [premise] is true; therefore, the major [premise] is false"[13].

The logical structure of Richard's argument could be symbolized in a modern system in this way:

$(\forall x)(Lx \supset \sim\sim Mx) \not\models L(\forall x)(\sim Mx \supset \sim Lx)$. This, in itself, is a curious claim since the conclusion is T derivable from the premise. Richard, however, rejects the inference in light of the syllogism counter-model, namely:

$\{L(\forall x)(\sim Mx \supset \sim Lx); \sim Ma\} \not\models L\sim La$.

To see that this formalized version of the claim entails a rejection of the axiom schema of S5, consider the following derivation:

{1} 1. $L(\forall x)(\sim Mx \supset \sim Lx)$	assumption
{2} 2. $\sim Ma$	assumption
{2} 3. $L\sim a$	interdefinability
{2} 4. $\sim a$	axiom of necessity
{2} 5. $M\sim a$	axiom of possibility
{2} 6. $LM\sim a$	modus ponens and S5 axiom schema
{2} 7. $L\sim La$	interdefinability

Q. E. D.

Clearly, if one accepts the axiom schema of S5, one must hold that $\{L(\forall x)(\sim Mx \supset \sim Lx); \sim Ma\} \models L\sim La$. Hence it follows that Richard must reject this axiom schema if he is aware of, or is presented with, it. Now indeed, Richard makes this argument as a possible objection to the quaestio which concerns the conversion of negative de necessario propositions. But in his reply to the objection, he seems to retain this portion of the larger argument as acceptable. If so, Richard must reject the axiom schema of S5.

This seems to contrast with Richard's attitude toward a proposition like the axiom schema of S4. He claims:

"Another principal argument: it does not follow of necessity: "Nothing impossible is that which is contingent to be impossible; therefore, nothing of necessity which

[13] RICHARD OF CAMPSALL, *Questiones super Librum Priorum Analeticorum*, in *The Works of Richard of Campsall,* Vol. 1, Edward A. Synan (Pontifical Institute of Mediaeval Studies, Toronto 1968), pp. 109-110.
"*Aliud principale: non sequitur de necessitate: nichil quod erit necessarium erit inpossibile, igitur, de necessitate nichil quod erit inpossibile erit necessarium, quia antecedens est verum et consequens falsum. Veritas antecedentis patet quia qu[a]libet singularis est vera et falsitas consequentis patet quia ex ipsa et alia vera sequitur unum falsum arguendo sic: de necessitate nullum inpossibile erit necessarium; te fuisse episcopum erit inpossibile; igitur, de necessitate non erit necessarium. Conclusio est falsa quia indeterminatum est utrum erit necessarium vel non, et minor vera, igitur maior falsa*".

is contingent to be impossible is impossible" because the antecedent is true and the consequent false"[14].

The interesting claim here is that, "Nothing impossible is that which is contingent to be impossible". Richard asserts it as true, and does not subsequently reject it in the reply to the objection (as he rejects the claim that the consequent is false). This could be expressible symbolically as (∀x)((M~Mx • M~~Mx) ⊃ ~~Mx). And to see that this formalized proposition entails the disjunction of the axiom schema of S4 and the axiom schema of S5, consider the following derivation:

{1} 1. (∀x)((M~Mx • M~~Mx) ⊃ ~~Mx)	assumption
{1} 2. (M~Ma • M~~Ma) ⊃ ~~Ma	instantiation
{1} 3. (M~Ma • M~~Ma) ⊃ Ma	double negation
{1} 4. (M~Ma • MMa) ⊃ Ma	double negation
{1} 5. ~(M~Ma • MMa) v Ma	def. of '⊃'
{1} 6. (~M~Ma v ~MMa) v Ma	de Morgan
{1} 7. ~M~Ma v (~MMa v Ma)	association
{1} 8. (~MMa v Ma) v ~M~Ma	commutation
{1} 9. ((~MMa v Ma) v ~M~Ma) v ~Ma	addition
{1} 10. (~MMa v Ma) v (~M~Ma v ~Ma)	association
{1} 11. (MMa ⊃ Ma) v (~M~Ma v ~Ma)	def. of '⊃'
{1} 12. (MMa ⊃ Ma) v (LMa v ~Ma)	interdefinability
{1} 13. (MMa ⊃ Ma) v (~Ma v LMa)	commutation
{1} 14. (MMa ⊃ Ma) v (Ma ⊃ LMa)	def. of '⊃'
Q. E. D.	

It is obvious that line 14 above is a disjunction of instances of the axiom schemata of S4 and S5 respectively. Hence, it seems that if Richard were to accept the formalized proposition (∀x)((M~Mx • M~~Mx) ⊃ ~~Mx), then it follows that he must accept one, the other, or both axiom schemata. We have argued above that Richard evidently must reject the axiom schema of S5 based on his espoused views. But if that is so, clearly he must accept the axiom schema of S4, again based on his espoused views. This is very significant theoretically since this would represent the earliest argument for acceptance of a proposition like the S4 axiom schema for which I have evidence[15].

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[14] *Ibidem*, p. 110.
"*Aliud principale: non sequitur de necessitate: nullum inpossibile est id quod contingit esse inpossibile, igitur nichil de necessitate quod contingit esse inpossibile est inpossibile, quia antecedens est verum et consequens falsum*".

[15] This is true at least for mainstream logicians. For a possible earlier acceptance of a proposition like the axiom schema of S4, and a definite earlier acceptance of a proposition like the axiom schema of S5, see below.

The first logician of whom I have record, explicitly accepting a proposition like the axiom schema of S5 is Ramon L'ull. L'ull was born ca.1232 probably in Spain[16]. He seems not to have received a formal education nor to have held a faculty position. He was a prolific writer who included among his logical writings *De Conversione Subiecti et Praedicati et Medii, Liber de Possibili et Impossibili, Liber de Fallaciis*, and many others too numerous to mention. He died ca. 1315.

L'ull was very much outside the mainstream of the logic theory of his day. He was interested in logic primarily as a tool for converting Moslems. Hence, it is easy to overlook his contribution to modal logic.

In his *Liber de Possibili et Impossibili* he says:

> "The subject of this book is necessity, since it is the genus of possibility and impossibility, seeing that what is possible necessarily is possible; and thus of the impossible. Otherwise, a contradiction would be implied"[17].

What does L'ull mean by this remark? Why would a rejection of the S5 schema or a proposition like it imply a contradiction? Well, it seems that L'ull posits a three tiered logic, the first of which is of interest to us. This first category of his modal logic looks much like the kind of modality (logical) we seek. Ramon states:

> "God is a necessary being, existing per se simply and infinitely. From which it necessarily follows that he is pure act. In such a being, the possible and impossible cannot penetrate. Otherwise, a contradiction would be implied, seeing that the possible lays it down that that which is in potency can be reduced to act, by reason of which the possible is called a positive habit. The impossible, though, which is a privative habit, lays it down that the possible cannot be. On account of which it is a privative habit. God, however, cannot be supposed to be subjected under these two habits since he himself is superior [to them] as cause and necessary being. Rather, he causes the possible and the impossible, so that with these he acts extrinsically. And therefore when it is said, "God cannot [do] this", whatever it be, it is understood that God cannot [do] this, because he wills this to be impossible. And when it is said, "God can [do] this, but wills not", it is understood that God can [do it] absolutely, be he wills this not to be decreed. Otherwise his will and other faculties would be subjected to the possible and impossible—which cannot be [the case]. Hence, since it is so, it is obvious for God to be exempted from the action and passion of the possible and impossible as a cause [is exempt] from its effect. And therefore those speak incorrectly who affirm that God cannot [do]

[16] Biographical material taken from KRETZMANN et al., op. cit., p. 879.

[17] R. L'ULL, *Liber de Possibili et Impossibili*, in *Corpus Christianorum Continuatio Mediaevalis XXXIII*, Helmut Riedlinger (Rpt. Turnholt: Brepols, 1978), p. 384.
"*Subiectum huius libri est necessitas, cum sit genus possibilitas et impossibilitas, quoniam quod est possibile, necessari[o] est possibile; et sic de impossibile; aliter implicaretur contradictio*".

> something, on the assumption that [that item] is according to the decree of God and [an object] of his cognition and service. For on such an assumption it follows that a being ab alio could resist [both] a being a se and first causes, which is impossible"[18].

From this passage it is clear that God, who is a necessary being, chooses what shall be possible and what impossible. Hence it follows, as per his earlier remark, whatever is possible is necessarily possible, and whatever is impossible, is necessarily impossible. Therefore, we seem to have the very first instance of a proposition like the axiom schema of S5, and consequently an entailment of a proposition like the axiom schema of S4 since it is contained in systems including S5.

IV. Sermoneta, Maior, De Soto and Fonseca

By the fifteenth and sixteenth centuries, mainstream logicians (many of whom were commentators on the fourteenth century masters) seem largely to have accepted propositions like the S5 axiom schema and to have adopted the S5 stance. Alexander Sermoneta, a fifteenth century Strode commentator, states the case for a proposition like the axiom schema of S5 thus:

> "The third doubt [is] whether that which is contingent necessarily is contingent, understanding by 'contingent' the complex significate. But the significate [of] contingent I call what can, or could, be or not be indifferently. It shall be replied by laying down four propositions. The first is: "If it is possible that Socrates runs, it is necessary that it is possible that Socrates runs." And I speak concerning the possible as contradictorily opposed to the non-possible or the impossible *per se*. For that is called impossible *per se* which neither can nor could be. So, converse-

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[18] R. L'ULL, op. cit., pp. 385-386.
"*Deus est ens necessarium, per se existens simpliciter et infinite; ex quo necessari[o] sequitur, quod sit actus purus. In tali ente possibile et impossibile intrare non possunt; aliter implicaretur contradicto; quoniam possibile ponit, quod illud, quod est in potentia, possit reduci ad actum, ratione cuius possibile dicitur habitus positivus. Impossibile vero, quod est habitus privativus, ponit, quod possibile non possit esse; propter quod est habitus privativus. Deus autem non potest poni pati sub istis duobus habitibus, cum ipse sit superius tamquam causa et ens necessarium. Sed causat possibile et impossibile, ut cum ipsis agat extrinsece. Et ideo quando dicitur: Deus non potest hoc, quidquid sit, intelligitur Deum non posse hoc, quia ipse vult hoc esse impossibile. Et quando dicitur: Deus potest hoc, sed non vult, intelligitur, quod Deus potest absolute, et vult hoc non esse ordinate. Aliter sua voluntas et aliae rationes paterentur sub possibili et impossibili; quod non potest esse. Unde cum ita sit, patet Deum esse denudatum ab actione et passione possibilis et impossibilis tamquam causa a suo effectu. Et ideo male dicunt illi, qui aiunt Deum non posse aliquid, posito quod sit secundum Deum ordinatum et ad eius notitiam et sevitium. Nam ad talem positionem sequitur, quod ens propter aliud resistere possit enti propter se et primae causae; quod est impossibile*".

> ly, that is called possible which can [be] or which could be. Then the proposition is proved, because let the opposite of the consequent be given along with the antecedent, namely, that, "It is possible that Socrates runs" and "It is not necessary that it is possible that Socrates runs." Therefore, "It is possible that it is not possible that Socrates runs." Therefore, "It is impossible that Socrates runs." The last inference holds because no complex significate is possible to be impossible unless the significate itself is impossible. Moreover, the necessary or the possible itself would be possible as it would be impossible, [both] which are absurd"[19].

The first proposition is clearly like the axiom schema of S5. His defense of this claim is a reductio ad absurdum:

> "Then the proposition is proved, because let the opposite of the consequent be given along with the antecedent, namely, that "It is possible that Socrates runs" and "It is not necessary that it is possible that Socrates runs. Therefore, "It is possible that it is not possible that Socrates runs". Therefore, "It is impossible that Socrates runs""[20].

Stated formally, the proof—with necessary adjustments—looks like this:

{1} 1. Mα	assumption
{2} 2. ~Lma	assumption
{2} 3. M~Mα	interdefinability
$\varnothing$ 4. M~M$\alpha \supset$ ~Mα	
{2} 5. ~Mα	modus ponens

Since lines 1 and 5 contradict, it is easy to see how one can derive the conclusion, LMα. Q. E. D.

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[19] *Ibidem*, f. 13v.
"*Tertium dubium utrum quod est contingens necessario sit contingens: intelligendo per contingens significatum complexum. Significatum autem contingens voco quod potest aut potuit indifferenter esse et non esse.*
Respondetur premittendo quatuor propositiones.
Primo est: Si possibile est Socratem currere necesse est quod sit possibile Socratem currere: loquor autem de possibili ut opponitur contradictorie non possibili seu impossibili per se. Dicitur enim impossibile per se quod nec potuit esse nec potest esse. Ita quod possibile dicitur per oppositum quod potest aut quod potuit esse. Tunc probatur propositio: quia detur oppositum consequentis cum antecedenti, scilicet, quod possibile est Socratem currere. Et non necesse est quod sit possibile Socratem currere. Ergo possibile est non esse possibile Socratem currere: ergo impossibile est quod Socrates currat. Tenet ultima consequentia, quia nullum significatum complexum possibile est esse impossibile nisi ipsum significatum sit impossibile: alioquin necessarium aut possibile ipsum possibile esset ut esset impossibile qu[a]e absurda sunt".

[20] Latin text found above.

Notice in the proof above that I have deliberately omitted the justification for line 4. Concerning this, Sermoneta says:

> "The last inference holds because no complex significate is possible to be impossible unless the significate itself is impossible"[21].

How can Alexander justify this claim? Notice that line 4 can be derived from the S5 axiom schema as follows:

{1} 1. $M\alpha \supset LM\alpha$	assumption
{1} 2. $\sim LM\alpha \supset \sim M\alpha$	transposition
{1} 3. $M\sim M\alpha \supset \sim M\alpha$	interdefinability
Q. E. D.	

It is likely, then, that Alexander was thinking about a principle like the axiom schema of S5 in order to justify line four.

As remarked earlier, logicians who adopted the S5 stance continued to proliferate in the sixteenth century. One such logician was John Maior. Maior seems to accept a proposition like the axiom schema of S5. His acceptance is inferred from the following remarks:

> "Every true divided modal [proposition] is necessary, and every false [divided modal proposition is] impossible"[22].

It is easy to see that the axiom schema of S5—$M\alpha \supset LM\alpha$—could be derived from this rule. Since it applies to every true modal proposition in divided sense, let α be a true modal proposition in divided sense. (It is understood that α will be a categorical proposition, i.e., either atomic, a negation, or a quantified proposition—namely an A, E, I, or O categorical proposition—but not a hypothetical proposition, i.e., a proposition formed recursively from two propositions and a binary connective. Maior's example of a true divided modal proposition is "Socrates possibly runs.") The mode in α will be either L or M. The interesting case is where the mode is M—as in Maior's example. Maior claims that α is necessary. What this means then, in his adduced example, is that "It is necessary that Socrates possibly runs" which is deduced from the truth of "Socrates possibly runs." If we reinterpret α as $M\alpha'$—where α' is 'Socrates runs', it is clear that his inference, in a formal system, amounts to $M\alpha' \supset LM\alpha'$. But this is just the axiom schema we want. So Maior's rule seems to entail the axiom schema of S5[23].

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[21] Latin text found above.

[22] J. MAIOR, *Introductorium perutile in Aristotelicam Dialecticen*, Paris. 1527. f. 67 v. "*Omnis modalis divisa vera est necessaria, et falsa impossibilis*".

[23] See COOMBS paper, *John Mai[o]r and Domingo de Soto on the Reduction of Iterated Modalities*, in I. ANGELELLI and A. D'ORS, *Estudios de Historia de la Logica*, Ediciones Eunate, Pamplona 1990, pp. 165-166.

On the other hand, it is interesting that Maior restricts this rule entailing the axiom schema to propositions in divided sense. Why does he do this? Another question is, Why does he hold this rule at all? I shall forego answering these questions until we examine the works and opinion of his student Domingo de Soto.

Domingo de Soto was born 1494[24]. He was educated at the University of Paris under Maior. He taught at the University of Salamanca. Among his major logical works are his, *Summulae*, and *In Dialecticam Aristotelis Commentarii*. He died 1560.

De Soto's S5 stance is more extensive than Maior's as evidenced by the following two propositions:

> "Every divided modal [proposition] is necessary or impossible"[25].

and:

> "Every composite modal [proposition] is either impossible or necessary, just as the divided [modal propositions]"[26].

It is interesting to note that de Soto, in these two remarks, accepts a proposition entailing the S5 axiom schema without qualification. Why does he do this? To answer this question we must first examine his understanding of the modal terms. He claims that:

> "For the solution of the argument it is to be noted [that] these four modes—necessarily, possibly, etc.— are accepted in two ways, namely, naturally and simply (or by whatever power). The naturally possible is that which can be by natural causes, as the coming into being of things. The [naturally] impossible [is] that which cannot be by natural causes, as the creation or passing away of the heavens. The simply possible is that which God absolutely can make to be, as the creation of another world. The simply impossible is that which implies a contradiction, as for a man to be a horse. The naturally necessary is that which cannot be otherwise by natural causes, as the heavens and the intelligences. And the [naturally] contingent [is] that which can be or not be through natural causes, as for Peter to sit. But the simply necessary is that which can be otherwise by no cause, as that God exists. And the [simply] contingent [is] that which can be or not be by divine power, as for Peter to be blessed. Therefore, to the minor [premise] of the argument we reply that the modal [proposition] is contingent now, as is obvious

[24] Biographical material is taken from KRETZMANN et al., op. cit., p. 859, and ASHWORTH, op. cit., p. 290.

[25] D. DE SOTO, *Summulae*, Salamanca 1554. f. 74v.
"*Omnis modalis divisa est necessaria vel impossibilis*".

[26] *Ibidem*.
"*Omnis modalis composita vel est impossibilis vel necessaria sicut divisa*".

> through its assertion, although at some time it shall be naturally necessary. For that which is simply necessary, as God is, always is necessary. But, however, that which now is contingent can be made to become naturally necessary. For after the creation of the soul of Antichrist, through no natural cause can it be made to become so that it is not"[27].

It is remarkable that de Soto's naturally/simply possible, etc. distinction approximates almost perfectly the contemporary distinction physically/logically possible, etc. The only difference being that de Soto equates the essence of a logical contradiction with God's absolute power to actualize a certain state of affairs, whereas a contemporary analysis would usually be content merely to note such a contradiction, and claim that its actualiztion is not imaginable.

But once de Soto resorts to talk of the simply possible, he must make an ontological move to match it—specifically he must make room for some possible entities. Indeed, in this way Dr. Jeffrey Coombs accounts for de Soto's acceptance of the S5 axiom schema—or a rule entailing it—both with respect to composite modal propositions and divided ones. Coombs claims:

> "The basic difference between [de] Soto's and Mai[o]r's interpretation of the term 'possible' in composed modal propositions of this sort can now be succintly stated. For Mai[o]r, "possible" stands for an actual proposition while [de] Soto believes it stands for a possible individual"[28].

To understand why Maior's view commits him to deny the axiom schema of S5 with respect to composite modal propositions while de Soto's does not, recall that, for most mediaeval logicians, propositions must be written, spoken or

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[27] *Ibidem*, ff. 79v-80r.
"*Pro solutione argumenti notandum, hos quatuor modos, necessario, possibiliter, etc. dupliciter accipi, scilicet, naturaliter et simpliciter, seu per quancunque potentiam. Possibile naturaliter est, quod per causas naturales potest esse, ut generationes rerum. Impossibile, quod per causas naturales non potest esse, ut productio vel corruptio coeli. Possibile simpliciter est, quod Deus absolute potest facere, ut creatio alterius mundi. Impossibile simpliciter est, quod implicat contradictionem, ut hominem esse equum. Necessarium naturaliter est quod per causas naturales non potest aliter esse, ut coelum et intelligentiae. Et contingens quod per causas naturales potest esse et non esse, ut Petrum sedere. Sed necessarium simpliciter est quod per nullum causam potest aliter esse, ut Deus esse, et contingens quod per potentiam divinam potest esse et non esse, ut Petrum esse beatum. Igitur ad minorem argumenti respondemus quod illa modalis est contingens modo, ut patet per suam de inesse, licet aliquando erit necessaria naturaliter. Illa enim quae est simpliciter necessaria, sicut Deus est, semper est necessaria. Sed tamen naturaliter potest quod nunc est contingens, effici necessarium. Nam post productionem animae Antichristi, per nullas causas naturales potest fieri ut non sit*".

[28] J. Coombs, *John Mai[o]r and Domingo de Soto on the Reduction of Iterated Modalities*, in I. Angelelli and A. d'Ors, *Estudios de Historia de la Lógica*, Eunate, Pamplona 1990, pp. 177-178.

thought about. But if propositions are the entities in question in composite modal propositions, then clearly, even when their subject matter is necessary, they can fail to exist due to either: 1) not being written down (or being erased); 2) not being spoken at all (or the puff of air, which is the sentence, evaporates); or, 3) not being thought at all (or the mind forgetting). Hence, a composite modal proposition may contain the mode possible, as:

(P) That Socrates runs is possible.

Nevertheless, when thought about as Maior does, it is clear that (P) is not necessary since it can cease to exist in one of the three ways described above.

Notice, however, that this is not a problem for Maior with respect to divided modal propositions since he regards modal terms in these as standing for first intentions and hence either: I) syncategorematic, modifying the copula; or, II) categorematic, and thus describing the modal status of the entity in question. So in "Socrates possibly runs", "possibly", taken syncategorematically, modifies the copula 'is', (which is understood, since "Socrates possibly runs" is elliptical for "Socrates possibly is running"). But taken categorematically, it means that Socrates' running is a possible being. Hence there is no restriction on the necessity of true divided modal propositions for Maior.

On the other hand, since de Soto is thinking about possible beings which God could actualize through his absolute power, then if a proposition states, say, a possible state of affairs, it must be possible on the grounds that God has the possible objects in his mind. Then, however, it follows that God necessarily has these possibles in his mind, since to hold otherwise would commit one to the view that God could destroy some of his mental contents. Hence, if any proposition—divided or composite—reflects a possible state of affairs, that state of affairs must be necessarily possible[29]. Hence, his acceptance of a proposition like the S5 axiom schema. One further scholastic logician who adopts an S5 stance is Pedro de Fonseca.

Pedro de Fonseca was born ca. 1528, probably in Portugal[30]. He was educated at and taught at the University of Coimbra. Among his major logical works is his *Institutionum Dialecticarum Libri Octo*. He died 1599.

The following remarks are pertinent to an S5 stance:

> "For example, all these are necessary, "That a man is an animal is necessary"; "For a man to be a grammarian is contingent"; "For a man to die is possible"; "For a man to be a stone is impossible" because they are true in such a way that they can never be false. These, indeed, "That a man be an animal is not necessary"; "For a man to be a grammarian is not contingent"; "For a man to die is not possible"; "For a man to be a stone is not impossible", these, I say, are impossible because

[29] For further discussion of this, see COOMBS, op. cit., pp. 177-180.

[30] Biographical material is taken from A. DUMITRIU, *History of Logic*, vol II., trans. Duilu Zamfirescu, Dinu Giurcaneanu, and Doina Doneaud. Kent: Abacus, 1977, p. 205.

> they are false in such a way that they can never be true. Since, therefore, all modal [propositions] which can be produced are either true in this way or false in the other way, it is not to be doubted but that all [modal propositions] are necessary or impossible. So especially the matter of modal [propositions] is either necessary or impossible, though not contingent. From which it follows, as from a foundation, that we mentioned briefly above, that every modal proposition is necessary or impossible"[31].

One wishes that Pedro had explained the "in such a way" addition above. Does he mean only that, for example, "For a man to be a stone is impossible" because "No man is a stone" is true by definition? And so the proposition with the modal word is necessary because it cannot fail to be true? Could one then continually assert of the previous modal proposition that it too is necessary? A reasonable conjecture is that Pedro adopts the view of his contemporaries and predecessors and identifies the possibles as existing in God's mind.

V. Effects on the early modern conception

If my understanding of the development of S5 stance and its subsequent predominance in the fourteenth through sixteenth centuries is correct, it becomes rather obvious why a Leibniz in the seventeenth century would formulate the postulates of theodicy with a view to all possible worlds.

Moreover, if my understanding of the ontological underpinning of propositions entailing the axiom schema of S5 posited by late mediaeval and second scholastic logicians (namely, the necessity of God's having all the possibles in his mind) is correct, it again becomes rather obvious why a Descartes would want the possibles to be such, not because God finds them somewhere, but because he makes them to be such, that is, willingly contains them in his mind.

And in making this last observation, we address the problem hinted at in the prologue of this paper—how to provide a coherent ontology for the axiom schema of S5. And so in positing God's mind as a sort of domain for the possibles, we can—as L'ull does—force those possibles (if you will) to be necessarily such.

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[31] P. DE FONSECA, *Institutionum Dialecticarum Libri Octo* 1611, Book III ff. 87-89.
"*Exempli causa, hae omnes sunt necessariae. Hominem esse animal est necesse, Hominem esse grammaticum est contingens, Hominem mori esse possibile, Hominem esse lapidem esse impossibile, quia ita sunt verae, ut falsae numquam esse possint. Hae vero, Hominem esse animal non est necessae. Hominem esse grammaticum non est contingens, Hominem mori non est possibile, Hominem esse lapidem non est impossibile, quia ita sunt falsae, ut verae numquam esse valeant. Cum igitur omnes modales, quae efferri possunt, aut illo modo sint verae, aut hoc modo falsae, dubium non est, quin omnes sint necessariae, aut impossibiles... ut materia modalium quae praecipve aut sint necessaria aut impossibilis, nulla vero contingens. Ex qua velut radice nascitur id, quod paulo superius diximus, omnes modalem enunciationem esse necessarium, aut impossibilem*".