Constantin Antonopoulos*

THE QUANTUM LOGIC OF ZENO: MISCONCEPTIONS AND RESTORATIONS

1. Continuity and the Law of Contradiction

Consider the following syllogism:

1. When a body is at rest, it is only where it is and nowhere else but there.

2. Motion and Rest are contraries all around.

3. Therefore, by force of (1) and (2), when something moves, it is not the case that it is only where it is, and nowhere else but there.

4. Therefore, by force of (3), when something moves, it is not only where it is but also where it is not.

5. Nothing can be where it is not.

6. Therefore, nothing moves.

This is one way among many of stating Zeno's objection to motion. Suppose we wish to avoid his conclusion by attacking the premises of the syllogism. Since (2) is a tautology, there remains only premise (1). But attacking (1) would only make matters worse. For then a body would not be (only) where it is even when at rest. And that is surely no antidote.

The last option remaining is to then question whether (3) is validly drawn from the contrariety between motion and rest, stated in (2). The opposition between motion and rest may be concentrated *elsewhere*; not in the description given. For example, "dead" and "unconscious" are also contraries. But an unconscious man will often be mistaken for dead because, despite the contrariety, these two states share a lot of properties in common: lying down, not moving, not reacting to stimuli, e.t.c. Only when it comes to breathing, will they be antithetically distinguished. They are contraries but not contraries *all around*. So it may be with motion and rest. Then the rest would not follow.

But it isn't. Motion and rest *are* contraries all around, at least in all respects of relevance to the argument. Suppose that (3) is not validly derived from (1) and (2) and, therefore, that it is false. If so, then a moving body will not be where it is not, but only where it is. But this will make motion and rest look *identical*, not contraries, and this contradicts the premises of the argument, that is

* Dept. of Applied Mathematics and Physics, National Technical University of Athens.

«acta philosophica» · ii, 16, 2007 · pp. 265-284

to say, (2). With the description of motion given in (3) –and (4)– withdrawn, motion and rest would just collapse on one another and be literally indistinguishable, in sheer absence of any *other* criterion of demarkation. And then surely nothing would move. The motion-rest contrast is *saturated*. It spreads all across the definitive board. Hence the conclusion is inevitable.

That something which moves cannot be only where it is, but has (somehow) to be also where it is not, is no news to any one. Velocity cannot be defined *at* a space point even in classical mechanics; at least two successive points are required for its definition, which, as we shall see later on, reaches to the very heart of Zeno's paradoxes in Hooker's understanding of them; (which will also be my own). Motion on a *point* is simply no motion. Hence what moves *has* to be where it is and (somehow) also where it is not. Dilemmas of this sort had made Hegel particularly happy, and had formed a significant part of his Dialectics: "Zeno, who first showed the contradiction native to motion concluded that there is no motion." [Hegel, § 89, 133.] But to Hegel this was but another instance of things which "they are and they are not". [*Ibidem*]

I have extensively argued [Antonopoulos, 2003] that Zeno's paradoxes are paradoxes of infinite *proximity*. I will not repeat the point made. I will instead present a brief but telling argument not previously included. Zeno's infamous runner stares at the 400m ahead of him and cannot lift a leg to cross it, standing paralyzed at its starting line. If he could take the *first* step there is, the first ever, then all the rest would follow upon it in natural procession. But, there *is* no first step to take [See Barnes, 1982, 262], if distances are infinitely divisible and hence such as are constituted of infinitely *proximally* packed points. The first step 'ahead' in conditions of infinite proximity of any point with any other would be right where he is standing. Hence, the runner cannot move.

To realize the inescapability of this argument, just consider asking someone to start counting numbers from the beginning. He will say "one, two, three, four ..." and so on. No problem. Now ask him to start counting, again from the beginning, but to count *fractional* numbers instead. He cannot. There just is no place from which to start. It is a logically impossible task to count fractional numbers from the beginning. He will remain as speechless as Zeno's runner is said to have remained motionless. There is no first fraction to start from and what cannot *start,* cannot *be.* [1] This is equally true for Zeno's run-

[1] Atomism seems the sole escape from this riddle, for it furnishes the all too requisite start, that is to say, a *foundation*. And I say "seems" rather than "is" because Atomism is itself a very spurious foundation indeed, as Zeno's brilliance has already anticipated in his paradox of extension: "How can a line of finite length be divided into infinitely many parts of finite length? And how can a line made up of lengthless parts add up to a line which has length?" [Harrison, 1996, 273]. Of course, this is Kant's Second Antinomy, between the Composite and the Simple. The Simple, *qua* Simple, ergo *structureless,* cannot be a compo-

ner, if the structure of distances to start crossing coincides with the structure of fractionals to start counting. He has nowhere to *step* on, hence no way to move.

All in all, when two states which we initially regard as successive are subsequently shown by deep level analysis to be infinitely proximal, there *is* no succession, motion itself included under the term. This conclusion, as I have already made clear, I consider warranted, but not exhaustive, for it leaves one particular option unmentioned, if it is an option. What, say, if these two infinitely proximal states are contradictories? Is it still true that even so nothing changes? Clearly not. For if –A is infinitely proximal to A, that is to say, indistinguishable from A, then there will be change of the sort which Zeno denies but Hegel asserts. If a thing 'eo ipso' contains its negation, the primordial law of Hegelian dialectics, change will in fact occur and, strangely, occur on Zeno's own terms. For then a thing will no longer have to abandon the state it is in in order to advance on to the next. If a moving body can be where (yet) it is not, it will be able to move, even if the two points are infinitely proximal. For now, in a different connection, it will be at a point where it was not. And so at another point. Or if a state can be what (yet) it is not, the state will be able to change, even if the two states are infinitely proximal. For now, in a different connection, it will be in a state in which it was not, and so in another state.

That this is the last option for the sake of salvaging motion, I share Zeno's sentiments down to their last letter. Were the matter to stand so, I too would conclude that nothing moves. But others, and not just Hegel (see below), do not.

Where there is continuity, there is no *nextness*. There will always be further points between points, or states between states, however closely packed. Hence transition to a different point or state is impossible (Zeno) unless the difference is already *there* (Hegel). And this sits poorly with LNC. If changes are continuous (i.e. infinitely proximal), and if changes also are transitions from a certain state A to a certain state –A, changes can only occur contradictorily, if *nothing* is to stand in-between the initial state and the final for keeping them apart, as the notion of uninterrupted continuity entails. And if something is, what can that something be? A further state?

Well, no. The problem will only reemerge by tentatively introducing further and further substates contained inbetween A and –A for setting them

nent of the Composite! [See Antonopoulos, 48, 2004, for a lengthy discussion of this point.] But perhaps this is not as bad as it seems. Foundations are not supposed to participate in the structures which they underpin, to begin with. For they would then stand in need of a foundation themselves. Whence, presumably, Leibniz's famous aphorism that "monads have no windows to enter from or depart". [*Monadology,* prop. 7.]

apart, however many, since, generally, *any* state other than A is in this connection an –A and will therefore reintroduce the same conditions of unhealthy proximity, only getting less and less healthier as subdivisions inbetween A and –A's keep piling. Hence, contradiction is avoided only if *something* is to stand inbetween A and –A, preventing their contact. The difference is, however, that as matters presently stand, this "something" cannot be a *state*.

Let us focus on the spatial analogue of this point. Either "here" and "not-here" are infinitely proximal, whereupon we but reobtain Zeno's initial contradiction in that exact same form, or else *non*-proximal, therefore separate, therefore at a distance. But if at a distance, what can *exist* inbetween the distance? There is but one answer to this question, in simultaneous satisfaction of all requirements: Nothing. For if anything of like nature is inserted, i.e. a "not-here$_1$" inbetween "here" and "not-here", the initial distance between "here" and "not-here" is just shortened. But its nature is always the same: "Here" will not be *distinguished* from "not-here", just because infinitely many other "not-here's" are inserted between them. If anything, then it is the very contrary. This is *the* way to draw "here" and "not-here" that much closer and all the way down to unhealthy infinite proximity. So it has to be *nothing* inbetween, no other *place* allowed, if these two are to be sharply distinguished – or a contradiction; (discontinuity already announces itself. *And* its role.)

Continuity of motion or change therefore create trouble for the LNC or, to put the point modestly, at least *some* kind of trouble, considering that most mathematicians tend to dismiss Zeno's paradoxes, as if just due to a bad mistake. Still, mathematicians have to do *something* about that mistake, in fact a great deal [See Harrison 1996; or McLauglhin & Miller, 1992], which they wouldn't have to do in its absence and this is trouble enough; what was intuitively warranted before now takes a lot of *argument,* if it is to be established.

Both warrings camps, the Zenonian and the Hegelian, are in harmonious agreement on this: Continuous motion keeps poor company with LNC. Only their *meta*-logical choices are at variance. Zeno endorses the Law of Contradi-ction and sacrifices motion, Hegel endorses motion itself, and sacrifices LNC, and by extension, therefore, taking sides with continuity, though the latter postulate comes up trivial (or absent) in his system when compared with the far more radical enterprise of denying LNC. One does not go looking for ways to mend the hole in one's dinner jacket when beneath it one is shot in the heart.

But what Hegel adopts only implicitly, or just idly on the face of it, namely, continuity of processes, modern day commentators on his behalf, especially G. H. von Wright, support explicitly: Continuity involves contradiction or, at best, cases open to a dialectical account (perhaps suitably mitigated by von Wright's own nonbivalent Truth Logic). And then the idleness in Hegel's system will turn out very much active indeed. When continuity is withdrawn, Dialectic is hardly as 'compelling' as it would have seemed prior to withdrawal, to those at least to whom it might have seemed compelling in the first place.

That all continuous change, especially continuous motion, is ultimately at variance with LNC is, it should not go unmentioned, Zeno's primary and private discovery. Its importance is multi-faceted and multi-dimensional, operating at once on many levels of epistemology. To start, it implicitly provides the clues to searching for alternatives capable of breaking free from both horns of the riddle; how to warrant motion (or change) *and* rescue LNC at the same time. If continuity is what is responsible for the clash of motion with LNC then what to look for is all but spelled out, isn't it?

Secondly – and this connects with my specific problem at hand in a variety of ways – Classical Mechanics (CM), the epitome of continuity, turns out hardly as classical as certain people (e.g. Einsteinians) are quick to presume and, accordingly, Quantum Mechanics (QM) hardly as incomplete too. If motion can only occur by a moving body being where it is not, and this is proven an immediate consequence of (classical!) continuity, it is clearly absurd to demand of the sole physical theory which *avoids* this, QM, to be more 'complete' than presently available, in its current *separation* and recognition of incompatibility of a $\Delta p=0$ with a $\Delta q=0$. What would Einsteinians have us do to secure comple-teness? Contradict ourselves? I can hardly overemphasize my indebtedness to C. A. Hooker for opening my eyes on this matter twenty three years ago on just how classical CM really is: «Momentum is formally defined in terms of a limiting process between two distinct positions, at least *classically,* since it is a derivative of position. This means that within strict logic, momentum is not defined *at* a place but only over an *interval*. One should certainly not accept the consistency of the classical description merely on the face value of its traditional use as consistent». [*Private correspondence*, dated 15 August 1983.][2]

Einstein's obsession with completeness and his correlative dismissal of Complementarity are all in futile defense of a chimera: That a system in motion can consistently have both a sharp momentum, $\Delta p=0$, and a sharp position, $\Delta q=0$, with nothing amiss either way. It can, if LNC is first sacrificed for the sake of 'completeness', now attained at the cost of having the body move in a point! Zeno's voice resounds in modern day scientific ears from three millennia ago: «What is moving moves either in the place in which it is or in the place in which it is not. And it moves neither in the place in which it is nor in the place in which it is not». [Epiphanius, *Adversus Haereticos,* quoted in Barnes 1982, 276.]

[2] In Section 2 we will retrieve more recent statements of the point in question, when the special relevance of the "(δ) *neighbourhood*" of the point, over which velocity is defined, is considered, fully confirming Zeno's initial insights into the matter.

Let me elucidate the point. If the arrow *moves* in the place where it is, then it is not in the place where it is. And nothing can be not in the place where it is. And if it moves in the place where it is *not*, then the arrow is in the place where it is not. And nothing can be in the place where it is not. Hence, nothing moves, at least not in this way. I would take quantum discontinuity any day to escape this problem *and* warrant the possibility of motion at the same time, however weird (conceptually, mind you, but not logically) discontinuous motion may turn out to be, and however 'incomplete' the description of processes subject to its limitations. One can always choose classical completeness instead, and for Δp, $\Delta q = 0$ at one and the same time, literally have the body move in a *point*. Hegel has already done it once and restless logicians toying with inconsistent logics do it all the time. [Below.] But at least they are conscious of what Einsteinians -and Einstein- were unaware of, when they demanded this kind of completeness.

Having come to this, no alternative can be *that* bad. So let's suppose, if only for the sake of argument, that things do not after all move as continuously as all that and see what follows. The body will then traverse the distance from a certain point A to a certain point B *without* traversing over any other, intermediate point. Then, while on its way from A to B the body would be nowhere at all in-between A and B. This must be so, for if anywhere inbetween them, and then anywhere in-between them and that, and so on, the body would just move continuously, contradicting the supposition. So it must be nowhere in-between.

Now I would say that, if something is nowhere at *all* inbetween A and B, it certainly cannot be at *two* places at once in-between A and B. For if the body is nowhere at *all* inbetween them, then the last thing it can do, is to be at *two* places at once inbetween them. And if it cannot be at two places at once in-between them, it cannot be where it is not. It will at all times be only where it is, when it *is*, and be nowhere at all, when it is *not*. This is exactly what an electron does, when traveling from one quantized orbit to another. [See Gavroglou, 1989, 551; Antonopoulos, 2003, 507-8.] This is certainly incomplete and probably a lot worse, but it is at least consistent.

2. LNC Vis A Vis the Law of the Excluded Middle

There may have been various motives behind experimenting with inconsistent or 'paraconsistent' logics [von Wright, 1986, 5], other than the influence of Zeno's paradoxes, but the paradoxes were clearly decisive [below]. Nevertheless, alternative reasons for the endeavour are not lacking, as is A. S. Karpenko's conviction that "contradictions are *useful* essences" [1986, 63]. It is perhaps so, provided we don't go looking for them everywhere. The author in question, however, does. More alarmingly than thus far indicated, his interest in paraconsistent logic is not containable within a sharply demarcated region. Its essential drive is actually of a rather universal character.

It consists of "extracting a paraconsistent structure from within *many-valued* logic" [ibidem], which is a very different type of objective. It is now implied that nonbivalent logics are inherently inconsistent, which means that a mere suspension of the Law of the Excluded Middle (LEM) *eo ipso* harbours inconsistency. And this is what makes all the difference. Nonclassical systems suspending LEM, a routine by now, are claimed to sacrifice *LNC* on top of it as part of the initial deal. (A 'package deal'?) For whatever properties a logical system is found to display, it either displays them necessarily, or else they have no bussiness being there at all. Providing an answer to this tendency, which, as we shall see, is anything but a solitary phenomenon, will now have to be included as a top priority issue in the plan of designing the right sort of logic for handling Zeno's motion. The design will in addition have to take care of *that,* namely, the suspicion or the possibility that a violation of LEM is translatable into a concomitant violation of *LNC*.

But first things first; we must first assess the status of LNC itself vis a vis Zeno's peculiarities of motion and examine the pressures it exerts on the introduction of paraconsistent logics. How these are to be handled per se and whether or not *other* options, escaping inconsistency by one route, only result to inviting it through the back door just after (Karpenko's way), will depend on the answers given. At a primordial stage, the clash between motion and LNC is, I submit, inescapable: «Although the derivative can be used to represent speed at an instant, its relevance in explaining motion at that instant is problematic. In order to compute the derivative at a particular time T, we must be able to evaluate φ at all t in some – open – *neighborhood* of T. Thus, defining the motion of the arrow using the derivative at a particular time T requires knowledge about the arrow's position at *additional* instants of time; we must know its motion throughout some *interval* containing T». [Alper, Bridger, 1997, 146].

In other words, if instants of time *additional* to T are necessary for computing the arrow's speed at T, then space *points* additional to the point occupied by the arrow at T are thereby necessary, hence points *other* than the point itself, where the arrow is *at* at T. Hence, in this process of determining the arrow's speed at T the arrow is taken as (clasically!) being where it in fact is not. The arrow's speed is literally *ahead* of the arrow! And if its speed is, the arrow is. The Zenonian analysis prevails. Hence, according to Hooker's earlier formulation, «Velocity cannot, strictly speaking, be defined *at* a point location but only in some (δ) *neighbourhood* of the point. Essentially, this point was already recognised by Zeno over two millennia ago and formed an important part of his famous paradoxes. Conceptually, the logic of the situation is clear: precise positions strictly preclude velocities and precise velocities strictly preclude precise position descriptions» [Hooker, 1973, 188].

[For an identical point see also McLaughlin&Miller, 1992, 373.] Now I can understand as well as the next man how this perplexing situation may tempt logicians to go looking for answers in violation of the LNC. What I cannot understand at all is the following 'extension' of it: «A thesis is put forward, call it θ. It has an antithesis which is its negation $-\theta$. It is then shown, one way or another, that the thesis is not true. It is also shown that the antithesis is not true. Thus *neither* the thesis *nor* the antithesis is true. From this is concluded (??) that both the thesis and the antithesis *are* true. This is called Dialectical Synthesis. To illustrate, the arrow in Zeno's antinomy is neither moving nor at rest at a given point of its trajectory. Therefore (??) it is both moving and at rest» [von Wright, 1986, 5; dark italics for the author's emphasis. Question marks mine.]

(For the original, Hegelian passage, see [Hegel, 1975, 126-7.) One wonders to what extent the contents of this astonishing passage relate at all to those of its next of kin, Karpenko's identical 'discovery' of how LNC is affected, if only LEM is denied. Other than that, the passage is an unprecedented scenery of fallacies, contradictions and circularities(!), all in a hotly boiling, undifferentiated cauldron. First, we are told that *neither* the thesis *nor* the antithesis are true. Then we are told in no uncertain terms that from this "is concluded(?) that *both* the the thesis and the antihtesis *are* true" (italicized in the original). But if *neither* is true, how can they be *both* true? Well, there is indeed one way and one way only to 'do' this, even if it is both, contradictory to the assumption *and* circular in one single move. The way is to take the 'thesis' and the 'antithesis' as the *sole* existing alternatives, i.e. A and –A. Then, presumably, if –A is "not true", then, by total elimination, we are left with A. Hence, A. At the same time however since A itself is also said to be "not true", –A. Hence, A and –A.

But this is contradictory to the assumption that neither A *nor* –A, which, if adhered to, would imply that LEM is transgressed here because there are now options *other* than A and –A alone. And hence that in their face -A *cannot* be turned into A by applying Double Negation, LEM's identical twin. This is what "neither A nor –A" *means*. And, apart from contradicting the assumption, the move is also circular on top of it, for to adopt Double Negation, when LEM has been shown (or simply assumed) false, is no longer an option, except circularly. (It is truly beyond me that *Synthese* has published this paper.)

For myself, I can only say this: If a pair of contradictory assertions are both *false* (or, if it be preferred, "not true" [3]), then the last thing that can ever

[3] I insist on this redundancy to forestall ironical remarks of the sort "the author conflates between LEM and the -so called- Principle of Bivalence", Łukasiewicz's contrivance to smuggle through his first system of three-valued logic with as little resistance as possible. With von Wright's Truth operator, T, in front of a variable we can indeed distinguish

follow from this as a premise is that that pair of contradictory assertions are both *true*. If they are the former, then the last thing in the universe that they can then be, is the latter. This should be straightforward enough. But there is a realization of much wider significance contained in the point. A violation of LEM eo ipso satisfies LNC. By the book and by definition. Logicians have not realized this all too important fact. If a pair of contradictory assertions are both false (or "not true"), thus transgressing LEM, LNC is eo ipso validated. For LNC would be at peril, if they were both true instead. Namely, the last thing they can ever be, if they are both false (or "not true" – it makes no difference really). Karpenko and von Wright have got the truth upside down. No three-valued logic thus designed may consistently turn out 'paraconsistent'.[4]

Properly speaking, no three-valued logic ever can, at least on the purely formal level (namely, if no semantic content is ascribed to the variables), unless of course, LEM and LNC are assumed *interderivative* laws. Now Karpenko, possibly, and von Wright definitely, do present cases justifying this conclusion, for they both affirm that, when LEM goes, LNC as a *consequence* goes too, though how they view this as a consequence of anything is frankly beyond me. On the level of an uninterpreted formalism this simply cannot be done unless, I repeat, LEM and LNC are interderivative. But are they?

The truth is that in two-valued logic they are indeed treated as interderivative, in the sense that, on two-valued standards, a denial of LEM is as 'self-contradictory' as the denial of any tautology comparable with LNC itself. The 'demonstration' of this point, a text book favourite, goes like this:

between the two: "Tp v T-p" is Principle of Bivalence, "Tp v –Tp" is LEM, in von Wright's four-valued system. The first says "*p* is either true or *false*". The second, "*p* is either true or *not-true",* making room for *dispersive* negations of *p,* e.g. ¬p, ¬p, ¯p, &c, in opposition to the classical, nondispersive negation, -p (or von Wright's T-p), which thereby limits the possibilities to two. Quite so; the difference is that LEM is the law of the excluded *middle,* and in this mutilated form, "Tp v –Tp" it excludes no middles. And the difference is that LEM is a *law*, and laws have to exclude to *be* laws, whereas in this mutilated form it excludes nothing. Logicians who insist on distinguishing between LEM and Bivalence are apparently oblivious of the fact that, if LEM ever intended to preclude something, this was precisely the possibility of *dispersion* of negation.

[4] There is indeed a way of showing that LNC and LEM may fail simultaneously in a nonbivalent system, depending on operational rules, and I mention it to avoid lethal confusion. This can occur if LNC and LEM are sweepingly violated in a system, and violated in *that* order. Namely, if LNC goes *first* (e.g. as in taxonomic dilemmas). Then A, in being 'both' true-and-false will, trivially, be neither-true-nor-false in opposition to other propositions which, in not being the two former, will also not be the two latter. A lacks a *definite* truth value, for it can accomodate *both*. If this is what von Wright is trying to say to us, he has simply got it backwards. One *can* get to a (derivative) violation of LEM *via* a prior violation of LNC. But the *converse* of this is anything but valid. It is, in fact, a contradiction.

1. –(P v –P)	Provisional Assumption
2. P	Assumption
3. P v –P	2, V Introduction
4. –(P v –P) & (P v –P)	3, 1 & Introduction
5. –P	2, 4, Red. Ad Absurdum
6. P v –P	5, V Introduction
7. –(P v -P) & (P v –P)	1,6 & Introduction
8. --(P v –P)	5, 7 Red. Ad Absurdum
9. P v –P	8, Double Negation

[Lemmon, 1971, 52].

What is remarkable in this 'proof' is that there is not one single step of the syllogism which is not circular. First of all, assumption of P at step 2 is intentionally inconsistent with Assumption n.1, whose present status contains *–P* as a partial premise. To then assume P at 2 is to force a contradiction where otherwise none may exist, only to proceed and derive a ...contradiction because of it.

Secondly, the rule of "Vel Introduction", permitting us to connect disjunctively any variable with its own negation, e.g. A v –A, though a two-valued tautology, is clearly *circular* in the face of Assumption n.1. Unless –(P v –P) is *independently* refuted, the possibilities are more than two, hence Pv–P rather than a tautautology is a falsehood, now that they may be *both* false and a third possibility, e.g. ¬P be the case instead. So this move is a circular non-sequitur. And as if all this isn't enough, when a 'contradiction' is derived on Step 4, entailing the rejection of the first alternative, P, Lemmon introduces *–P* in its stead, as if this is the only option in existence, once P is ruled out. But other options always are, insofar as Assumption n.1 remains (independently) unrefuted. And in its face one cannot conclude that –P, if P is ruled out, as in Step 5, and derive the second contradiction, by *again* circularly applying the rule of "Vel Introduction". Finally, self-contradiction and circularity are once again casually adopted in step 8, where --(P v –P) is triumphantlly turned into P v –P, as if it is not *this* above all else which the yet unrefuted premise –(P v –P) forbids.

And the less complex, but far more pregnant 'proof', presented below, demonstrates that the job of deriving a contradiction by a provisional denial of LEM cannot be done except circularly. In this case by adopting the rule of Double Negation for deriving P from –P, as shown in step 3. LEM is not a tautology except circularly and hence, since LNC is a tautology, the two laws are wholly independent and can never be reduced to one another (i.e. LEM to LNC.)

1. –(P v –P)	Ass.
2. –P & --P	1, Eq.
3. P	2, DN
4. P & –P	2,3 & Int.

The usual (incorrigible) reply I have been receiving, when I raise the previous objections to the (so-called) 'demonstrations' of LEM, is that these demonstrations are perfectly valid in two-valued PC, where Double Negation is a valid rule of inference, validly employed in these demonstrations. I will only say this and say no more. LEM *is* DN, viz. LEM=DN. So when you assume –(P v –P) as a premise, you are assuming –DN as a premise, *eo ipso*. If one is blind to this tautology, I'm not the right person to consult. Consider yet another argument, less formal than the previous, but formal enough to matter:

LEM: A proposition is either true or false and nothing else is possible.

Is this proposition a tautology? If so, then what of the proposition which follows?

LEM1: A proposition is either true or false and nothing else is possible, *if and only if truth and falsehood are the sole existing possibilities*.

It is the latter statement, LEM1, which is the real tautology as, I trust, the italicized, *added* section has made evident. In consequence, LEM itself is not, for it lacks precisely this crucial ifclause which makes LEM1 one. And trying to assimilate LEM1 into LEM will only introduce another circle. Whether or not truth and falsehood *are* the sole existing possibilities, which provision alone would assimilate LEM1 into LEM, is precisely the point at issue. Logicians who have supposed LEM to be a self evident law, equivalent with LNC, have a lot to answer for.

However, those of them who have supposed the contrary, i.e. Hegel, von Wright and Karpenko, have a lot more. These three assume, on the one hand, that LEM is not a valid law, which it isn't, and then feel free to dispute it but then, all too strangely, they dispute LNC together with it, within one and the same process of reasoning, *as if these two laws were equivalent*. I am at a loss. The sole reason for treating these two laws as equivalent, is only if they are both *as* self evident. Whereupon their denial would be an equal absurdity and then they'd have to observe both rather than deny both. They do *not* wish to consider them as self-evident, so that they may proceed to deny both? Fine. But then, if neither is self evident, the sole reason for treating them as equivalent is automatically removed. If *neither* of LEM, LNC is self evident, *whence* is inferred their equivalence? And if not equivalent after all, why suppose at all that they both go down together? LEM might and yet LNC might not. This is all just too confused.

The 'equivalence' between LEM and LNC is an (outdated) logician's fiction, at the expense of the former, of course. Let me then suggest how they may even *conflict,* now that this is established. Suppose A and –A are at the same time. This immediately conflicts with LNC. Does it conflict with LEM? Far from it. LEM would be at peril, if A and –A were both *false,* not if they are both true. (This is but a reversal of the initial point.) If A and –A are both true, LEM's disjunction, A v –A, is *a fortiori* true, since minimally satisfied even if only one of the disjuncts is true. Rather than being equivalent, LEM and the LNC are actually mutually *exclusive* for suitable value ascriptions: For A=false, –A= false, LEM goes, LNC stays. And for A=true, –A=true, LNC goes, LEM stays.

True enough, both pairs of ascriptions are downright two-valued PC impossibilities, allowing LEM and LNC to remain fully compatible within a PC context. But that is all behind us now. Within a nonbivalent context, introduced not by me but by Karpenko and von Wright themselves (and, esssentially, by Hegel), the situation is literally irrecognizable by two-valued standards. Yet neither Karpenko nor von Wright will consent to play by their own rules, continuing to treat LEM and LNC as equivalent, which, apart from having shown to be invalid even in 2-valued PC, is in addition overtly false in the nonbivalent context which is, after all, of their very own making.

To go looking for a paraconsistent structure "inside of many valued logic" is to commit two sins with a single act; it is to take LEM and LNC as equivalent, which they clearly are not even on 2-valued standards, and to disregard that, on nonbivalent standards, they are *antithetic*. A sin committed by von Wright to the exact same extent, when he complacently postulates that both the thesis and the antithesis *are* true (with italics), because actually "it is shown" that *neither* of them is! Besides being a blatant contradiction in its own right, this supposition still operates on an assumed LEM, LNC equivalence, necessary for turning the violation of the former into one of the latter, when such equivalence is only circular in bivalent PC and logically false in nonbivalent logics.

My own account at least keeps a steady course. LEM and LNC are not equivalent in the slightest in bivalent PC and so are on their way to being mutually *exclusive* in nonbivalent PCs. Which means that, contrary to what von Wright and Karpenko affirm, nonbivalent logics have to be consistent ***qua*** nonbivalent. Only thus will the axiom-cum-theorem "Both false, ergo none *true*" find its proper expression. And I select LNC at the expense of LEM *because* they are not equivalent in two-valued PC, LNC essentially being the sole true tautology between the two of them. And hence the only one that abides. Which selection is straightforward order of logical priority; the primeval concept, the tautological, abides, whatever may befall the subordinate, *quasi*-tautological one. In the end, the discovery that what befalls the subordinate, "A, –A both false", *satisfies* the primeval, "A, –A neither *true,"* is simply but a sign of its self-contained superiority.

To sum up: Failure of LEM is succes for LNC. How, then, does this link with Zeno's problem? Von Wright almost has the answer, before proceeding to turn multiply incoherent. He says the arrow "is neither moving nor at rest" at a certain point of its trajectory. And since, as repeatedly stressed, "neither moving nor at rest" cannot *ex hypothesi* be twisted into "both moving and at rest", we are left with just "neither moving nor at rest", which, though three-valued, is also consistent. There is a problem to be settled, however. The actual details of Zeno's argument do not show this. They show the contrary. Namely, that the arrow's motion directly assaults LNC itself; not LEM. It does, if its motion is *continuous*, but not otherwise. And its motion does not have to be continuous at all. In fact, it has to be the contrary, if continuity is what's behind the inconsistency. For only then will "here" and "not-here" be (infinitely) proximal.

However if, as already remarked, "here" and "not-here" are *not* proximal, in order to avoid immediate contradiction, they can only be *separate,* admitting of nothing inbetween. I.e. no *place* inbetween them for the moving object to *be*. And if the object is in *no place* inbetween those two, it obviously cannot be at *two* places inbetween those two. And therefore cannot be where it is not. Its motion is discontinuous. And once it is, "here" and "not-here" now set *radically* apart, the threat to LNC is thereby removed twice over. Once because "here" and "not-here" no longer touch, twice because, via discontinuity, what cannot be anywhere at *all,* cannot be at two places at once, either. However, LEM *has* been transgressed in this process. Even v.Wright has conceded that much, the sole point of substance in his passage. But let us not just take his word for it.

3. LEM, LNC and Discontinuous Transitions

Bertrand Russell, in paying his respects where respects are due, in this case to Zeno, proposes the following modification of our concept of motion, to make it compatible with his paradoxes: «People used to think that when a thing moves it must be in a *state* of motion. This we now know to be a mistake. When a body moves, all that can be said is that it is in one place at one time and in another at another». [1917, 65.]

This definition has discontinuity built into it. If *all* that can be said is that the thing is now at A, now at B, but not in a *state* of motion, while progressing from A to B, then it is now at A, now at B, and nowhere in-between, since *being* in a state of motion while advancing from A *to* B would automatically have to *place* it inbetween A and B.

Russell's definition connects fully with my own requirements. Suppose that there can be no place inbetween boundary points A ("here") and B ("not-here" – so "there"). If the moving body has left A but not *yet* emerged at B, we cannot say "the body has moved", for in absence of any other place for it to be, except B, (complete) motion will have not resulted *unless* the body emerges *at* B. Russell's Zenonian definition prescribes "...in *another* place at another (time)". Since this has not yet been fulfilled, we cannot say that the body has moved.

However, by the same token, though the body may not yet be at B, it is, nevertheless, no *longer* at A either. And in no longer being at the place, where it previously was, the body cannot be said *not* to have moved either. The definition of Rest prescribes "*being* at one place at one time", and this is no longer the case either. In consequence, and exactly as von Wright has perceived, the body is currently neither moving nor at rest. LEM is transgressed. However, at the very same time: [a] LNC is upheld. Since A (here) and B (not-here) are now separate, there is no question of a "here/not-here" proximity and therefore no threat to LNC. [b] The process is discontinuous. Since A and B are set radically apart (for fear of a contact between "here" and "not-here"), they eo ipso admit of no *place* inbetween them for the body to be. [c] If the body can be at *no* place, it cannot be at *two* places, and so cannot be where it's not.[5] [d] LNC is satisfied *because* the process is discontinuous. And, conversely, [e], the process is discontinuous *in order* that LNC be satisfied. The mutual coherence between the axioms and subordinate theorems extends all across the requirement board. All are parts of a unitary whole with no misfit involved.

The quantum instantiation of the point satisfies identical requirements:

P1 Time is a continuum.

P2 No physical system can be in two discrete states at the same time. I.e. there is a first instant, at which the succeeding state is occupied, and a last instant at which the preceding state is occupied.

Lemma: The energy transition must take a finite amount of time.

P^3 Energy exchanges involve finite, indivisible quanta of energy only.

Lemma: During the finite transition time no definite energy can be assigned to the system. [Hooker, 1971, 263. The Postulates are seven in the original.]

This is a typical model of a quantized transition. The energy indefiniteness argued for is due to the absence of any *smaller* values, other than the limiting one involved, resulting to the absence of any *other* values, as the energy decreases throughout the finite and continuous amount of transition time. So there is none left to assign to the system during this time.

Notice, first, the *complementary* structure of the phenomenon. Any narrowing of the total transition time *t* will immediately lead us directly into the "danger" area, during which the energy conservation theorem can no longer be upheld. The temporal placing of the event can be made more accurate only at the expense of an unambiguous definition of its energy, whence ΔE. Conversely, an unambiguous definition of the energy cannot but rely on whatever *definite*[6] energies are available in the process, namely, the initial and the final. But to achieve this, we must make the temporal placing of the transition reciprocally *wide* to include them both, so now Δt. As Hooker elegantly remarks, «to assign either the initial or the final energies

[5] Other than that, von Wright 'sees' a *contradiction* (correctly!) in describing the phenomenon as "neither motion nor rest". How he manages to see *that,* I guess I'll never fathom.

[6] The two energies, the initial and the final *are* definite. Otherwise their difference would not be a quantum, no more no less, as the theory demands.

would take one *outside* the transition itself: and because the quantum of energy is indivisible, there is no *intermediate* energy consistently assignable». [Hooker, 1972, 245.]

All in all, what the preceding (brief) analysis has shown is that the energy conservation theorem and the capacity to trace the system's evolution in time can never be jointly satisfied than to within the limits of a (minimal) energy value E=the quantized difference of the two states, changing *over* the interval *t*, i. e. no better than the product Et, which in this case has the value of the quantum of action. Hence, any description of the phenomenon in terms of the parameters of energy and time will display inaccuracies at best be equal, or, if not, then greater than the value of the quantum of action. Therefore, $\Delta E \Delta t \geq h$.

This, then, is the physics of the situation. But what about its logic? Take the transition from E=1 quantum, to the ground state, E=0. There is, Hooker explains, a last instant, at which E=1 is occupied, and, accordingly, a first instant at which E=0 is occupied. Which instants, as P^2 demands, are to be held apart.

Now take any instant such that it is always *later* than the last instant, when E=1, and such that it is always *earlier* than the first instant, when E=0. Since taken as later than E=1, the last positive energy value available, the system at that instant cannot be said to have any energy. However, since taken as earlier than E=0, the first instant when the system will cease having a positive energy value, the system at that instant cannot be said to *not* have any energy either. It follows, therefore, that LEM is violated in this model in a manner quite analogous to the one I have alluded to the arrow's motion, when occurring from one discrete place to another. And this analogy matters, for in Hooker's model of discrete energy changes, LEM fails if *LNC is first satisfied. This being none other than Hooker's P2 itself:* No system can be in two discrete states at the same time (and the rest).

LNC demands that the two contradictory states, "it has energy", P, and "it does not have energy", –P, be separated by a time t>0. So we interject a period *t* between t1, the last instant when still P, and t2, the first instant, when –P. So, t1<t<t2. Since now t1≠t2, LNC is satisfied. But then again, what is now the *case* at *t*? Since *t* begins *later* than t1, the last instant that still P, P is no *longer* true. And since *t* ends *earlier* than t2, the first instant that –P, –P is not *yet* true. Hence, during *t* neither P nor –P are true. The period whose interjection satisfies LNC is *eo ipso* a period which violates LEM, *because* of the interjection. In Hooker's model for quantized transitions, LNC and LEM turn mutually exclusive, as I have had occasion to repeatedly stress. A feature which is the constant companion of any transition devised along lines operating under similar, limiting conditions. In other words, in the case of discontinuous motion no less, subject to the same restrictions.

There is but one further item in my list of requirements left pending in Hooker's account of discontinuous transitions: I have stressed from the start, that it is a violation of LEM, which entails satisfaction of LNC. But Hooker's model has not exactly done that. In fact, it has done the converse. It has postulated a prior satisfaction of *LNC,* resulting to a violation of LEM. But this is certainly close enough! For if my central claim is that Zeno's paradoxes can be answered with discontinuous motion *and* if Hooker's model shows that during discontinuous transitions to retain LNC *is* to sacrifice LEM, then whatever nonbivalent logic is likely to result from this sacrifice, it will be by definition consistent. Which is precisely my argument also. So to integrate the point: to violate LEM in conditions matching the previous just insert a time t>0 between the *last* instant that P and the *first* instant that –P. Then there will be instants *after* the truth of P and *before* the truth of –P. This violates LEM, for during those instants neither P nor –P. But it *eo ipso* satisfies LNC because a period of duration t>0 has just been *inserted* between the last instant that still P, and the first instant that –P, forbidding their *contact*. Hence, almost trivially one is tempted to conclude at this point, to deny LEM is to assert LNC.

The explanation of this logical phenomenon is to be found in the corresponding structure of the *physical* phenomenon. Namely, discontinuity. During a discontinuous change there can be *nothing* admitted in-between the (discrete) boundary states, the initial and the final, in consistency with the premises. But if *nothing* is the case, then any definite description of 'nothing' will be false, a pair of any two contradictory assertions, P and –P, included. So LEM goes. However, for the very same reason LNC is satisfied. For if *nothing* is true of the case, nothing *contradictory* can be true of it either. Let us compare this with von Wright's account of a transition which is *not* discontinuous: «Consider a process such as rainfall. It does not stop suddenly, let us assume, but gradually. During a certain stretch of time it is first definitely raining, *p,* later definitely not raining, *-p,* and between these two states in time there is a "zone of transition", when too few drops may be falling to make us say that it is raining but too many to prevent us from saying that the rain has definitely stopped. In this zone the proposition that *p,* is *neither true nor false*. One could, however, also take the view that as long as some drops of rain are falling it is *still* raining –but also take the view that, when there are only a few drops of rain falling, then it is *no longer* raining. Then, instead of saying that it is neither raining nor not-raining, one would say that it is both raining and not-raining in this area». [von Wright, 1986, 12-3. His italics.]

So, by the same token, when the moving body is no longer at point A but not yet at point B, we "could also take the view" that it is both moving and not moving "in this area". And this is a view I have *not* taken. For then not only would we have failed miserably to confront Zeno, since the moving object *would* be where it is not. But, on top of it all, we would also have violated

LEM *and* LNC *and* shown motion to be impossible, at least on Zeno's standards. Some accomplishment this is for a logic intended to handle the paradoxes! But would we say what the author says we would? Not really. In fact, this conclusion of his is possible only on the admittance of the circularities and the contradictions already exposed in page 7, of which the passage quoted above is, allegedly, the 'illustration'. –P (not-raining) and --P (not not-raining) can be turned into P (raining) and –P (not-raining), only if --P is circularly and contradictorily turned into P. Hence, fortunately, we do not have "to take that view".[7]

Discontinous transitions of the sort examined here, i.e. Hooker's, violate LEM *on condition* that LNC is previously satisfied. Of which truth I have just demonstrated also the converse. There can therefore be no question whatsoever regarding their consistency. This much at least we may safely postulate, minimizing the army of problems, which Zeno's analysis of motion has disclosed and QT made official. In the face of this definite option, when one "takes the view" that an object is neither moving nor not-moving during the limiting interval, or that it neither has nor has-not any energy, during the corresponding, limiting interval, one *cannot* also "take the view" that the object is *both* moving and not-moving, or that it *both* has and has-not energy, during this interval.

We now realize that Zeno's dilemma, "what moves is *either* in the place in which it is *or* in the place where it is not" [p. 5], though valid, is not exhaustive. For it leaves one option unmentioned: What moves is *nowhere at all* (while moving). And therefore cannot be where it is not. Though conceptually this situation is utterly unmanageable, as are most, if not all, cases violating LEM, offerring no utilizable epistemological alternative to Zeno's impasse, *logically* at least it is as satisfactory as any nonbivalent logic is expected to be. Actually, more than most. For, as we have seen, this type of nonbivalent logic itself results, if at all, out of a prior, explicit satisfaction of the Law of Non Contradiction, concretizing my central contention here, that if a pair of contradictory assertions are both false, violating LEM, the last thing they can ever be are both true, violating LNC. If they are both false, LNC is *protected*, not threatened.

Indeed, von Wright's contention that we are at freedom to say the one as much as say the other is simply due to a common fallacy. The fallacy of mistaking a necessary condition for a sufficient one. It is all a question of criteria, really. To say truthfully of something that it is an X, it must be no less than X and, at the same time, no more than X. But the discontinuous transition from A to B, until completed, is less than motion, for the body has not yet emerged

[7] Would one take the view that "it is both raining and not-raining" when it is "neither raining, nor not-raining"? The people *I* know would just say it's drizzling.

at B. And if it is less than motion, it is not true that the body has moved. On the other hand, however, this discontinuous transition, once under way, is also more than rest, for the body is no longer at A either. And if more than rest, it is not true that the body has *not* moved either. Which two, if taken together, simply yield that neither one is true. And therefore anything but *both* true.

The argument, in other words, which seeks to turn "neither A, nor –A" into "both A and –A", is simply an argument mistaking an incomplete concept for a complete one.The discontinuous transition, once under way but not yet over, is incomplete motion and therefore, to the same extent, also incomplete rest. And therefore neither motion, nor rest. It is simply, a borderline case, where *both* wholescale alternatives, motion and rest, are equally ruled out. That's all.

4. The Man I'll Never Catch

It is time to recapture the entire line of reasoning which has led to the present result, and time to show why, besides being inevitable, it may also be, to a degree, desirable. Let us designate "Motion Is Continuous" as A. Let us also designate "Nothing Moves" as B. Then the following conditional can be reconstructed out of Zeno's reasoning, including his tacit premises:

[1] If motion is continuous, then either LNC is false or else nothing moves. In symbols, A→ –LNC v B. But LNC is true, hence nothing moves. In symbols, –LNC v B, *and* LNC; ⊢ B.

[2] But it is false that nothing moves; therefore –B. And, on the other hand, LNC itself is true. In consequence, –LNC v B is as a *whole* false. In symbols, LNC & –B. In consequence, – (–LNC v B).

[3] Hence, by Modus Tollens applied to [1] and [2], it follows that "Motion Is Continuous" is also false. In symbols, A→ –LNC v B, *and* –(–LNC v B);⊢ –A.

[4] Therefore motion is not continuous, i.e. –A. But if motion is not continuous, LEM is violated. In symbols, –A→ –LEM.

[5] But, as formerly demonstrated (several times over), when LEM is false, LNC is true. In symbols, –LEM→LNC.

[6] Consequently, by Modus Ponens successively applied to [3], [4] and [5], if motion is not continuous, then LEM is violated, and when LEM is violated LNC is validated. In symbols, –A, –A→ –LEM, –LEM→LNC, ⊢ –A→LNC, and, finally, –A & LNC. Motion is discontinuous and LNC satisfied.

This, in a nutshell, is the logic becoming to Zeno's motion –actually, to motion. And there is gain in this for general epistemology. Zeno's paradoxes show that continuous motion is impossible. Therefore, necessarily, either nothing moves or else motion is discontinuous. But things do move (the alternative unexplored by Zeno), hence motion is discontinuous. But discontinuous motion (and discontinuous change) necessarily violate LEM. And, in turn, the violation of LEM necessarily satisfies LNC. Therefore, if things do move, the Law of Contradiction is satisfied *on Zeno's standards*. This result, to my knowledge, is derived here for the first time. For it is derived from conceding Zeno's paradoxes and, so far as I can tell, concession of the paradoxes can only result to a two-edged dilemma: either motion occurs contradictorily or else nothing moves. But I have shown that concession of the paradoxes does not result to a dilemma. It results to motion in *satisfaction* of LNC.

I am overcome by the awsome challenge that, when I move, I have to be where I'm not. I'm not *that* fast. So I suppose I have to settle for being nowhere at all, when I move (between discrete, allowed places). Having to outrun one's self all the time is not the sort of competition one can live up to. Losing one's self now and then (inbetween the allowed points) is a far better rest to go to, even if it too has its price. Medieval philosophers may have dreaded the proverbial "horror vacui", but never lived long enough to see what was in store for us today. Compared to inconsistent logics – and many other odd peculiarities besides, to do the trick – Zeno's paradoxes are almost like paradise.

Having to be everywhere, just to go somewhere, is a strain far more demanding than being nowhere at all and still getting there. So this will have to be my choice. After all, if I'm at *no place* when I move, the last thing I need worry about, is being where I'm not, racing in vain after the man who is me. For if I'm at no place at *all* at a given time, I certainly am not at *two* places at that time. The trip may be slow and it sure will be rough. But at least I won't have to get there before I do.

ABSTRACT: *G. H. von Wright validly remarks that Zeno's arrow "is neither moving nor at rest". Then he invalidly proceeds to turn this into "both moving and at rest " eo ipso. Hegel does the exact same thing and so, it seems, does everybody else. A violation of the Law of the Excluded Middle (LEM), in the form of –A and --A, is equated (eo ipso) with a violation of the Law of Non-Contradiction (LNC), as A and –A. The move is both, circular (it employs double negation) and contradictory. When it is asserted that neither A nor –A, the last thing which follows is that both A and –A. If these two are both false, contradicting LEM, they cannot also be both true. Unexpect-edly, a violation of LEM protects LNC. In accordane [a] I argue that a system violating LEM eo ipso satisfies LNC, contrary to what von Wright, Hegel, paraconsistent logicians and everybody else seems to think. [b] Zeno's paradoxes produce an antinomy, iff motion is continuous. (Ancient Atomists only reacted to Zeno's infinite divisibility of processes.) [c] A comparable quantum model for discontinuous transitions, including motion, displays properties identical to those specified in [a] and [b]. If a body moves discontinuously from A to B, it is nowhere at all in-between A and B. And therefore cannot be where it is not, offerring an alternative to Zeno's antinomies. [d] Zeno's paradoxes, if handled by quantum discontinuity, lead to a 3-valued but consistent system. Discontinuity eliminates all possible descriptions of a system's state. Hence, if nothing can be truthfully said about the system during a discontinuous transition, nothing self-contraditory can be said about it either. Discontinuous motion is nonbivalent but consistent.*

References

1. J. S. Alper, & M. Bridger, *Mathematics, Models and Zeno's Paradoxes*, «Synthese», 110, 1997, pp. 143-166.
2. C. Antonopoulos, *From Zeno to Complementarity*, «Philosophy Today», 48 (1), 2004, pp. 63-86.
3. C. Antonopoulos, *The Tortoise Is Faster*, «Southern Journal of Philosophy», XLI (4), 2003, pp. 491-510. [Published one year later than Ref. Nr. 2.]
4. J. Barnes, *The Presocratic Philosophers*, Routledge & K.Paul, London 1982
5. K. Gavroglou, *Simplicity and Observability: When Are Particles Elementary?*, «Synthese», 79, 1989, pp. 543-557.
6. C. Harrison, *The Three Arrows of Zeno*, «Synthese», 107, 1996, pp. 271-292.
7. G. F. Hegel, *The Science of Logic*., Transl. William Wallace, Oxford 1975.
8. C. A. Hooker, *Energy and the Interpretation of Quantum Mechanics,* «Australasian Journal of Philosophy», 49, 1971, pp. 262-270.
9. C. A. Hooker, *The Nature of Quantum Mechanical Reality*, «Paradigms and Paradoxes», R. Colodny, ed., University of Pittsburg Press, 1972, pp. 67-302.
10. C. A. Hooker, *Metaphysics and Modern Physics*, in *Contemporary Research in the Foundations of Quantum Theory*, Reidel, Dordrecht 1973, pp. 101-303.
11. A. S. Karpenko, *Paraconsistent Structure Inside of Many Valued Logic,* «Synthese», 66 (1), 1986, pp. 63-69.
12. G. W. Leibniz, *Monadology*, Monroe Beardsley, ed., New York 1960.
13. E. J. Lemmon, *Beginning Logic*, Nelson's University Paperbacks, 1977.
14. W. McLaughlin, & S. Miller, *An Epistemological Use of Nonstandard Analysis to Answer Zeno's Objections to Motion*, «Synthese», 92, 1992, pp. 371-384.
15. B. Russell, *Mysticism and Logic*, London 1917.
16. G. H. von Wright, *Truth, Negation and Contradiction*, «Synthese», 66 (1), 1986, pp. 3-14.