# DOUBT ACCUMULATION AND THE EPISTEMIC VALIDITY OF LOGIC

Amir Horowitz*

Summary: *1. Multi premise closure. 2. Single premise closure. 3. Multi premise closure again.*

It seems natural to assume that one would be justified to believe whatever follows logically from what one justifiably believes. That is, deductive rules of inference preserve epistemic merits such as the justifiability or rationality of beliefs. This is the notion of the (logical or deductive) closure of justification, or, put otherwise, of the epistemic validity of logic.[1] While this notion seems obvious to many philosophers, various arguments against it have been suggested. Some of them aim at undermining the closure of justification under specific inference rules (notably *Modus Ponens* and Conjunction Introduction), and some of them aim at undermining it under any inference rule, or under any multi premise inference rule. I believe that the idea of doubt accumulation undermines this notion. This idea (the thought of which has been inspired by the preface paradox and the lottery paradox) underlies some discussions of the issue of multi premise inference rules, but surprisingly hardly any precise presentation of an argument against the closure of justification that is based on this idea has been suggested, and those arguments that have been suggested are not general. For example, Christensen takes this idea to directly undermine epistemic closure only as far as graded beliefs are concerned, and Lasonen-Aaranio's recent argument against epistemic closure is said to be effective only for an indeterministic world.[2] Relatedly, those argu-

* Department of History, Philosophy and Judaic Studies, The Open University of Israel, Ra'anana 808, 43107, Israel. E-mail: amirho@openu.ac.il. I am grateful to Levi Spectre and Yuval Eylon for helpful comments on an earlier version of this paper.

[1] The expression "epistemic validity" can be used to refer to the closure of *knowledge* too, but the present paper is concerned only with the closure of *justification* and this expression is used here to refer to it exclusively.

[2] See, respectively, D. Christensen, *Putting Logic in its Place*, Oxford University Press, Oxford 2004, and M. Lasoneb-Aaranio, *Single premise deduction and risk*, «Philosophical Studies», 141 (2008), pp. 157-173. Their arguments are directed against the closure of knowledge but it is clear that their considerations apply to the closure of knowledge *via* the closure of justification. Lasonen-AAranio takes her argument to be effective "at least" for an indeterministic world but I think it applies only to such a world.

ments are usually taken to apply only if evidential support and justification are understood in probabilistic terms. In the first part of this paper I suggest a general argument, one that is centered around the idea of doubt accumulation, to the effect that justification is closed under no multi remise inference rule. I will also show that considerations concerning doubt accumulation against epistemic closure need not be dependent upon a probabilistic understanding of evidential support, justification and doubt. The argument to be suggested is independent of such an understanding.

Some philosophers have suggested that the idea of doubt accumulation undermines also single premise closure of justification. In part II I argue that single premise closure can be said to founder on the rocks of doubt accumulation only under such an understanding of the closure that no one has ever wanted to endorse it. In part III I further pursue the significance of the failure of multi remise closure.

What makes the view that justification is deductively closed natural, is perhaps the following reasoning. In a deductively valid argument the conclusion is "contained" in the premises – there is nothing in the conclusion that does not inhere in the premises. Put differently, the conclusion is in no respect (logically) stronger than the union of the premises. Therefore, one who justifiably accepts the premises of such an argument would be justified to accept the conclusion. A simpler way to base epistemic validity on logical validity is to say that since we are justified to trust logic, we are also justified to trust whatever it is that logically follows from what we have reason to trust.

Indeed, we are justified to trust that logic would take us from true premises to true conclusions; this is what logical validity is about.[3] But we are not entitled to assume that what preserves truth preserves justification as well. Evidential support for a belief – that sometimes renders the belief justified – has a prominent feature that isn't shared by the belief's being true, namely that it is a matter of degree; it may be weaker or stronger (regardless of whether or not it renders the belief justified). The same idea may be expressed by saying that *doubt* comes in degree.[4] This idea is pre-

[3] The claim that logical validity preserves truth has been challenged, of course, but here I grant its truth. (There is a sense in which this claim is analytic, and then the challenge to the power of logic should be put in terms of questioning the possibility of valid logical rules.)

[4] I take the notion of evidential support to be unanalyzable. As the argument exemplifies, entities of various kinds may be said to endow beliefs or claims with evidential support. Facts may constitute evidence and thus support beliefs or claims; beliefs or claims (e.g., ones that concern evidence) or sets of beliefs or claims may support other beliefs or claims; inference and inference rules that are employed by arguments may endow beliefs or claims with evidential support. Evidential support is a sub-category of the category that I call "epistemic support" that includes also support by reasons that do not involve evidence.

supposed by some of the premises of the argument to be presented in part I.

## 1. Multi premise closure

Multi premise inference rules, in my use, are inference rules that essentially involve more than one premise. By saying that an inference rule *essentially involves* a certain premise I mean that this premise is necessary for the inference's warranting the truth of its conclusion. (Of course, the "official" inference rules do not include redundant premises, but I'd like to make a more general point, one that pertains to deductively valid arguments of all possible patterns.) The thesis for which I will argue, then, is that one may not be justified to believe in a proposition in virtue of the very fact that it follows according to any such an inference rule from what one justifiably believes. Formulations of the closure usually require that the subject have some epistemic attitude towards the inference they employ in order for justification to be preserved, e.g. that the subject *know* about the relevant entailment. In what follows I ignore this condition. It can easily be seen that taking it into account would not affect the argument.

Here is the argument:

1. Epistemic justification allows for doubt: A belief may be justified by evidence even though it is not conclusively supported by the evidence (that is, even though the proposition regarding the evidence does not entail this belief). [Premise]

2. There is a threshold (fuzzy as it may be) for justification: Evidential support for a belief may justify it, and evidential support for a belief may be too weak to justify it. [Premise]

3. Doubt accumulates (multi premise version): If one has inconclusive evidential support for one's belief in proposition p (whether or not this support is strong enough to make one justified in believing p) and one has inconclusive evidential support for one's belief in proposition q that has no logical or evidential connections with p (whether or not this support is strong enough to make one justified in believing q), [and… proposition r… and…] then one's evidential support for the union of p and q [and…] (that is, for p and q [and…] taken together) is weaker than one's evidential support for either of these propositions taken alone. [Premise]

Among the claims that may have such epistemic support, there are mathematical claims, perhaps moral claims, and others.

The claim that *beliefs* come in degree is not presupposed by the argument.

4. Logic does not generate evidential support *ex nihilo*: Any valid inference rule that essentially involves two (or more) premises endows the conclusion of any argument that is based on it with at most the same degree of evidential support that is had by the union of the argument's essential premises (i.e., by its essential premises taken together). [Premise]

5. Doubt may accumulates so as to ruin justification: The evidential support yielded for a belief by the union of two (or more) justified beliefs – a union of beliefs that would have justified this belief had it had the same evidential support as that of either of those (justified) beliefs – may fail to endow this belief with justification. [from 1, 2 and 3]

6. Any (valid) inference rule that essentially involves two (or more) premises may fail to provide one who justifiably believes each one of the essential premises of any argument that is based on this rule with justification for believing this argument's conclusion. [from 4 and 5]

The logic of this argument is straightforward, and I believe that its premises would be largely uncontroversial. Let me say a few words on their behalf. The first premise, that epistemic justification allows for doubt, should be accepted by anyone who rejects sweeping skepticism, since if justification endows beliefs with certainty, then most of our beliefs – specifically, our empirically-based ones – cannot be justified. Indeed, there are philosophers who reject this premise and would thus reject this argument, but it is important to note that philosophers who reject this premise would not attach much significance to the issue of whether logic preserves justification in the first place, since on their view justified beliefs are rare.

The second premise, that there is a threshold (fuzzy as it may be) for justification, is quite obvious. (Note that this premise presupposes the idea that evidential support for a belief comes in degree.) One way to realize the cogency of this premise is to notice that it follows directly from the (I believe undeniable) claim that evidential support for beliefs can fall short of justifying them. This premise is indifferent of whether the threshold for belief justification is "general" or context-dependent in some way. The supposed fuzziness and context-dependence of the justification threshold do not block the inference for the (intermediate) conclusion that doubt may accumulates *so as to ruin justification*. The idea might be that a conclusion of an argument might be such that its justification threshold is lower that those of the premises, and then the accumulation of the doubt of the premises may not be enough to undermine the justificatory status of the conclusion. This line of reasoning seems to me problematic in a few respects, but I'd be satisfied with pointing out that *the possibility* of the situation envisaged in no way clashes with my argument: for the argument to work, it suffices that cases in which valid arguments lead

reasoners from justified beliefs to unjustified ones be possible; it matters not whether there can be cases in which valid arguments lead reasoners from justified beliefs to justified beliefs. Having said this, let me emphasize that I think that there is *a large variety of cases* in which valid arguments do lead reasoners from justified beliefs to unjustified beliefs – see below.

Let's move to the premise that doubt accumulates. The evidential support for two (or more) uncertain independent propositions is weaker than the evidential support for each of them: the addition of an uncertain belief to one's uncertain belief whose uncertainty is independent of that of the former decreases the uncertainty – that is, increases the doubt – of one's stock of beliefs. (This premise too presupposes the idea that evidential support for a belief comes in degree.) This point would be well served by the following example.[5] Suppose that there are two semi-final matches in a football tournament. Suppose also that the facts about the competing teams` recent performance (and other relevant facts) support your belief that team A would win over team B and your belief that team C would win over team D, although the evidential support with which those facts provide you is (in each of the two cases) inconclusive and you should have some doubt in the truth of each prediction.[6] It seems clear that you should have stronger doubt in the prediction that team A and team C would be the finalists. And the more uncertain beliefs we add to this scenario (suppose that we are dealing with a lower stage of the tournament in which 32 teams play), the less evident would the "overarching" belief be. (Scenarios are possible in which your predictions regarding the two semi-finals are justified by the evidence yet it would be unjustified to believe that team A and team C would be the finalists. This claim is relevant also to other of the argument's premises.)

It may perhaps be easier to realize this idea if justification, evidential support and doubt are cashed out in terms of probability for the truth of beliefs, for it is easy to see that the probability one should assign to the event of team A and team C being the finalists is lower than the probability that one assigns to the winning of each one of these teams in the semi-final. But the example is described in terms of strength of doubt and does not appeal to probabilities. Now it is possible to argue that this talk of strength just hides the role of probabilities in the example, and that the idea that doubt accumulates prevails only if we understand it in such terms. However, the only possible ground for this claim is that justification (and doubt) can only be understood in terms

[5] I owe this example to Yuval Eylon.

[6] There are various reasons for doubting any prediction regarding football matches. One of them concerns the role of luck in football. See Y. Eylon and A. Horowitz, *What's Luck Got to do with it?*, in T. Richards (ed.), *Soccer and Philosophy, Beautiful Thoughts on the Beautiful Game*, Open Court, Chicago 2010, pp. 107-120.

of probability; and then, of course, for the argument that is presented here to work, the premise that doubt accumulates may be cashed out in terms of probability, and the example would support this premise even if it could only make sense if understood in such terms. So (assuming the other steps of the argument are effective) the argument seems to work independently of whether or not a probabilistic understanding of justification is correct.

The fourth premise, that any inference rule that essentially involves at least two premises endows the conclusion of any argument that is based on it with at most the same degree of evidential support that there is for the truth of all the essential premises taken together, is also obvious. For whence can further support come? It is possible to think that further support can come from the power of logic, but does it make sense to maintain that logic not only preserves degree of evidential support but also enhances it, that is, that it generates evidential support? Take, for example, the simplest (single premise) inference, that of "p therefore p". Certainly, this inference cannot endow p with any evidential support that it does not possess without it.[7] And this holds true with respect to any inference whose conclusion is logically equivalent to the union of its premises. But what if the conclusion is logically weaker than the union of the premises? This doesn't change the situation, since the extra logical content that the union of the premises possesses *relative to the conclusion* certainly need not support the conclusion. In short, whether or not the premises of an inference has extra logical content relative to its conclusion, the inference cannot endow the conclusion with evidential support beyond the evidential support that there is for the truth of all the essential premises taken together.

The argument's conclusion, then, is that any inference rule that essentially involves more than one premise does not preserve epistemic justification. Conjunction Introduction is the simplest inference rule that falls into this category of rules, but other inference rules, e.g. Modus Ponens, Modus Tollens and Disjunctive Syllogism also fall into it. Having two premises is enough for a deductive inference rule to fail to preserve epistemic justification; it need not involve great many premises, since doubt accumulation of even two beliefs may reach the level of ruining justification, if. e.g., one of them has evidential support just above the justification threshold.

The argument does not undermine the claim that a valid inference rule with one premise (such as Conjunction Elimination) preserves epistemic justification. Doubt accumulation of the sort that is described by the third premise is impossible for such inferences. The argument also does not affect the claim that a valid inference whose all premises are immune to doubt preserves epistemic justification, for trivially, when there is no doubt, no doubt accumu-

[7] I owe this point to Levi Spectre.

lates. (Of course, whether or not there are claims that are immune to doubt is a matter of dispute). In fact, the argument does not undermine the claim that a valid inference all but one of its premises are immune to doubt preserve epistemic justification, for in this case too doubt accumulation is impossible. So we may sum up the upshot of the argument in this way: any inference that essentially involves at least two premises that are not immune to doubt does not preserve justificatio.

## 2. Single premise closure

Some philosophers have argued that the notion of single premise epistemic closure is also untenable, and specifically, that it too founders on the rocks of doubt accumulation.[8] Indeed, appropriately adjusting the premise that doubt accumulates can make the argument that I have presented against multi premise closure an argument against single premise closure. That is, we have to replace premise 3 with:

3*. Doubt accumulates (single premise version): If one has inconclusive evidential support for one's belief in proposition p (whether or not this support is strong enough to make one justified in believing p) and inconclusive reasons to trust inference rule R (whether or not this support is strong enough to make one justified in trusting R), and one infers proposition q from p by means of R, then the epistemic support one has for q is weaker than the epistemic support one has for either p or R.

But unlike the break down of multi premise closure, the break down of single premise closure can only be interpreted as quite a trivial phenomenon. Note that premise 3* presupposes that single premise inference rules are not immune to doubt. This claim may be understood in terms of the possibility of errors on the part of individual reasoners in employing inference rules. We can certainly err in employing a chain of single premise inference rules, or in employing complex single premise inference rules (the "official" single premise inference rules are quite simple, but recall that our discussion is concerned with inferences of all possible patterns). Some of us, and under some conditions all of us, can err even in the single employment of a simple single premise inference rule. This possibility of error indicates that such reason-

[8] See K. DeRose, *Failures of single premise closure? The 'just barely' problem*, in *Certain Doubt*, June 29th, 2004, http://el-prod.baylor.edu/certain_doubts/?p=38, and note 14 of DeRose, *Introduction: responses to skepticism*, in K. DeRose & T. Warfield (eds.), *Skepticism: A Contemporary Reader*, Oxford University Press, Oxford 1999); M. Lasoneb-Aaranio, *Single premise deduction and risk*, in *ibidem*. It should be noted that both De Rose and Lasonen-Aaranio present their arguments as purporting to establish the conditional claim that if doubt accumulation undermines multi premise closure, it undermines single premise closure as well, although they do present arguments against multi premise closure.

ing is not immune to real doubt, and so there may be cases in which there is a real risk of doubt accumulation to the degree of ruining justification. But if all there is to the idea of the failure of single remise closure of justification is the idea that individual reasoners can err in applying such inference rules, I wonder who ever wanted to deny this idea. The idea of the closure of justification, when plausibly understood, is the idea that justification is preserved when inference rules are applied correctly from a purely logical point of view, or, in other words, that the *correct* logical application of inference rules is also epistemically correct. So the mere possibility of error in applying the rules is irrelevant to the real (or at any rate – the interesting) issue of the closure of justification.[9]

Now one might wish to base the failure of single premise closure not on the possibility of errors on the part of individual reasoners in applying inference rules but rather on the idea that, for some reason, inference rules cannot, in principle, be absolutely trusted. However, this too cannot be the issue, since the idea of logical closure simply presupposes logical validity. The question with respect to the logical closure of any feature is the question of whether logically *valid* inference rules preserve this feature, be this feature knowledge, justification, imagination, or whatever. To question validity is to change the issue. In sum, while doubt indeed can accumulate to the extent of ruining justification in the case of single premise arguments, this possibility does not clash with the view that justification is closed under single premise argument in its interesting and plausible sense.

## 3. Multi premise closure again

I conclude that under the real (or at any rate the interesting) understanding of the closure of justification, only multi premise closure of justification breaks down. This is important enough. Those inference rules that essentially involve only one premise are not very helpful in contexts other than "logical" ones. When we are trying to learn anything that isn't logic, in either mundane or highly theoretical matters, we (or at any rate ordinary people) rarely employ inference rules such as Conjunction Elimination or Disjunction Intro-

[9] The challenge of doubt accumulation to multi premise closure *would* challenge also single premise closure and avoid the difficulty I raise if we assume that inferences include a premise that expresses knowledge of (or justified belief in) the entailment in question. But there are good reasons to reject such an assumption, e.g. the one pointed out by Lewis Carroll in *What the Tortoise Said to Achilles*. Keith De Rose's claim that if doubt accumulation undermines multi premise closure it also undermines single premise closure presuppose such a knowledge condition; Ralph Wedgwood exposes this presupposition of De Roses's position and mentions the Lewis Carrol difficulty for it. De Rose concedes. See their exchange in *Certain Doubt*, June 29th, 2004, http://el-prod.baylor.edu/certain_doubts/?p=38.

duction (and even the more complicated single premise inferences are rarely employed in non-logical contexts). So the conclusion of the argument of part I is quite significant: it may be expressed by saying that all "truly instructive" inference rules do not preserve justification. Relatedly, the accumulation of the doubt associated with an argument's justified premises to the extent of ruining the argument's conclusion is by no means a marginal phenomenon. There is a large variety of such cases, for epistemic life isn't easy and many of our argument's premises are far from certain. Further, since many arguments that we commonly use involve more than two premises, cases in which doubt accumulates so as to ruin the justification of our conclusions are quite widespread.

It is not the moral of this argument that those valid inference rules that do not preserve justification according to it have no epistemic role to play, and specifically, that they cannot be used to take us from some justified beliefs to other justified beliefs. They can surely be used for this purpose on many occasions, for there are many cases in which the accumulation of doubt does not reach the level of ruining justification. Those rules, then, can be used for that purpose; they just have to be used wisely and cautiously, not automatically. In this respect, the epistemic utility of deduction is closer to that of induction than might first appear.

Abstract: *It seems natural to assume that deductive rules of inference preserve epistemic merits such as the justifiability or rationality of beliefs. This is the notion of the (logical or deductive) closure of justification. While this notion seems obvious to many philosophers, various arguments against it have been suggested. Some of them aim at undermining the closure of justification under specific inference rules (notably Modus Ponens and Conjunction Introduction), and some of them aim at undermining it under any inference rule, or under any multi premise inference rule. In part I of this paper I attempt to provide a precise presentation of an argument against the closure of justification under any multi premise inference rule. This argument is based on the idea of doubt accumulation. In part II I argue that, pace some philosophers, single premise closure can be said to founder on the rocks of doubt accumulation only under such an understanding of the closure that no one has ever wanted to endorse it. In part III I further pursue the significance of the failure of multi premise closure.*

Keywords: *Argumentation theory, Doubt Accumulation, Logic, Epistemic Validity.*