ON DISJUNCTION

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Aristotle by accepting the law of excluded middle:

ExM: pv~p,

while rejecting the law of bivalence:

BV: either p is true or ~p is true,[1]

denied that truth distributes over disjunction. For he held that BV implies fatalism, which he rejected, while he held that ExM is a necessary truth.[2] Aristotle thus repudiates the truth table account of disjunction.

Unexpected support for Aristotle's view comes from a standard theorem in the classical propositional logic. For contrary to the truth table account of disjunction, truth does not distribute in the theorem:

T: $[(p\&q) \supset r] \supset [(p \supset r) \vee (q \supset r)]$

on any of its standard readings. For the disjuncts in the consequent hypothesize that if either p or q is true then so is r, while the antecedent of T presumes only that if both p and q are true then so is r.[3]

One might think that the non distributivity of truth in T may be due to the

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¹ The paper assumes the equivalence between 'p is false' and '~ p is true'.

² Aristotle writes: «A sea-fight must either take place to-morrow or not, but it is not necessary that it should take place to-morrow, neither is it necessary that it should not take place, yet it is necessary that it either should or should not take place to-morrow… One of the two propositions in such instances must be true and the other false, but we cannot say determinately that this or that is false, but must leave the alternative undecided. One may indeed be more likely to be true than the other, but it cannot be either actually true or actually false» (Aristotle, *De Interpretatione* ch 9,19a 30-40).

«These awkward results and others of the same kind follow, if it is an irrefragable law that of every pair of contradictory propositions, […] one must be true and the other false,… that all that is or takes place is the outcome of necessity» (Aristotle, *De Interpretatione* ch 9,18 b:27-32). For a defense of Aristotle's position see A. Blum (2011).

³ That something was amiss was noticed at least since A. Blum (1986).

eccentricity of the material conditional in its consequent. But this is not so. For truth fails to distribute in:

T1: $[(p\&q) \supset r] \supset [\sim(p\&\sim r) \vee \sim(q\&\sim r)]$,

and:

T2: $[(p\&q) \supset r] \supset [(\sim p \vee r) \vee (\sim q \vee r)]$,

as well. For in both instances the antecedent requires the joint truth value of the antecedent for acquiring r, not so in the consequent.[4]

   T thus confirms Aristotle's thesis in a deeper way. For unlike in BV, in T neither disjunct need ever be true and yet the disjunction as a whole is true, if the antecedent of T is true.

   Fortunately, the difficulty to which T gives rise under any of the standard ways of understanding it by the truth table account is mitigated by the fact that neither disjunct in

$(p \supset r) \vee (q \supset r)$

detaches unless one of the disjuncts is denied. But the denial of one of the disjuncts will yield '$\sim$ r' and not 'r'. Hence neither disjunct by itself can consistently yield r.

References

ARISTOTLE (1941), *De Interpretatione* [*On Interpretation*], E.M. Edghill, trans., in R. McKeon, *The Basic Works of Aristotle*, Random House, New York 1941, pp. 38-61.

A. BLUM, $\supset$ *AND 'If…then…'*, «Logique et Analyse», 29 (1986), p. 49.

A. BLUM, *The Force of Truth*, «Philosophical Investigations», 34 (2011), pp. 393-5.

ABSTRACT: *We show that our basic logic, contrary to the truth table account, is in agreement with Aristotle that truth does not distribute over disjunction and thus as Aristotle contends the law of excluded middle does not imply the law of bivalence.*

KEYWORDS: *Aristotle, bivalence, disjunction, excluded middle, truth table.*

[4] In the antecedent of T1 what is not the case is that both p and q are true and r is false, but in the consequent '$\sim$ r' with either p alone or q alone is false. The antecedent of T2 in disjunctive form says that the disjunction consisting of r with the denial of both p and q is true while the consequent is satisfied with a disjunction consisting of r with either p alone or q alone.